Prof. Sumanta Hait

SACT,

Dept. Of Physics, Narajole Raj College

C4T (Wave Optics) , Topic :- Huygens Principle

INTRODUCTION

No one doubts that physics is an exact science. Nevertheless, the notion ’exact science’ “should not be interchanged with ’like mathematics’. As stressed by Huygens within physics, “one will find proofs of a kind, which do not grant the same great certainness of that of geometry and which even are rather different from those, because here, the principles are verified by the conclusions drawn from them, while the geometricians proof their theorems out of sure and unquestionable principles; the nature of the subjects dealt with conditions this”. Huygens’ ideas on how light propagates have become basic ingredients of our physical picture of the world. The notion Huygens’ principle (HP), however, is not uniquely used. This paper aims, on the one hand, at the clarification of some confusion existing in the literature, in particular, about the role of sharp, non-spreading wave fronts and the range of applicability. For instance, Feynman wrote, that HP holds exactly for wave mechanics, but only approximately for optics, and Scharf stated, that HP is a principle of geometrical optics, not of wave optics. On the contrary, the unifying power of HP will be demonstrated here.

Some of that confusion is related to Kirchhoff’s formula and reaches up to doubts on the validity of HP at all or on the possibility of the representation of HP by means of Green’s functions (GF). Both doubts contradict any mind believing in the unity of physics. Indeed, Kirchhoff’s solution to the wave equation, while being mathematically exact, suffers from the drawback of requiring the knowledge of both the field amplitude and its gradient on the boundaries. I will trace the origin of these mathematical and physical difficulties to the notions of degrees of freedom of motion and of independent dynamical variables. For the sake of the unity of physics, a further goal of this paper is to generalize Huygens' basic ideas. This means, that I will keep essentially the imagination, that each locus of a wave excites the local matter which reradiates a secondary wavelets, and all wavelets superpose to a new, resulting wave (the envelope of those wavelets), and so on. Huygens' *ad-hoc* omission of backward radiation as well as Fresnel's and other auxiliary assumptions is requested to be included in a natural manner. In particular, attention will be paid to a simple, but general and exact description of wave and other propagation processes, which obey the principle of action-by-proximity and can be described by explicit transport equations.

For historical and methodological reasons, I start in Section 2 with HP in mechanics and continue, in Section 3, with Kirchhoff's formula and certain problems of its physical interpretation. Then, Hadamard's rigorous definition of HP is discussed. In section 5, the superposition of secondary wave (let)s is represented and illustrated by means of general field propagators in the space-time domain. This leads to a description of wave motion, that overcomes the difficulties in the interpretation and application of Kirchhoff's formula mentioned above.

Wave Front

The wave front at any instant is defined as the locus of all the particles of the medium which are in the same state of vibration.

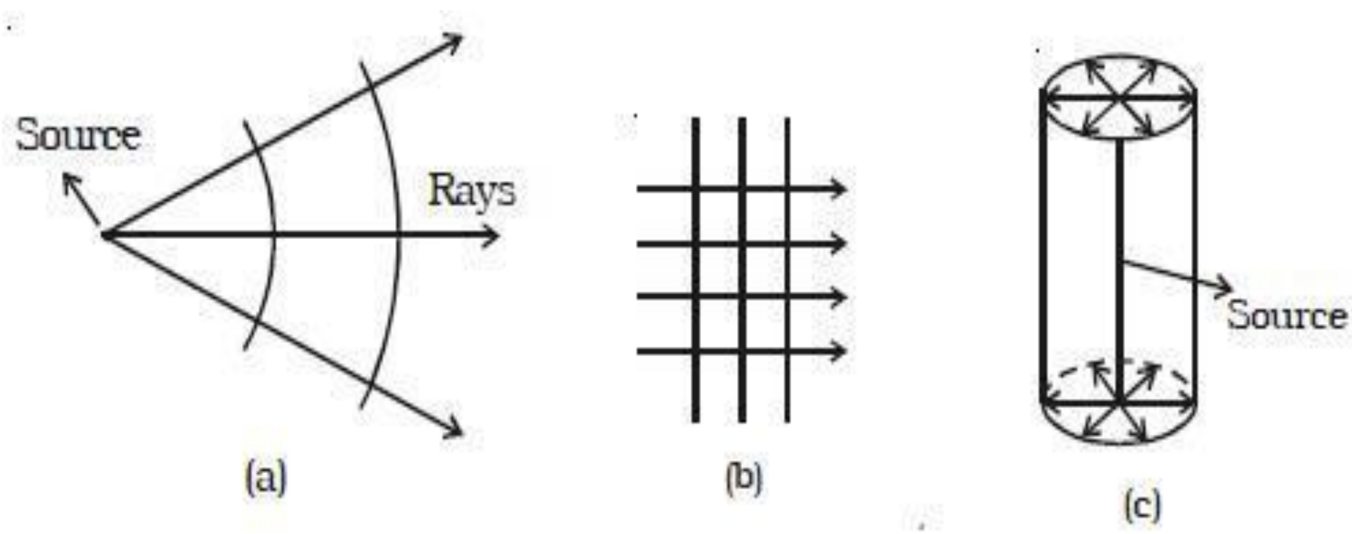
Or

An imaginary surface passing through particles oscillating with same phase is known as wavefront

A point source of light at a finite distance in an isotropic medium emits a spherical wave front (Fig a).

A point source of light in an isotropic medium at infinite distance will give rise to plane wavefront (Fig. b).

A linear source of light such as a slit illuminated by a lamp, will give rise to cylindrical wavefront (Fig c).



HUYGENS PRINCIPLE

Huygen’s principle states that,

(i) every point on a given wave front may be considered as a source of secondary wavelets which spread out with the speed of light in that medium and

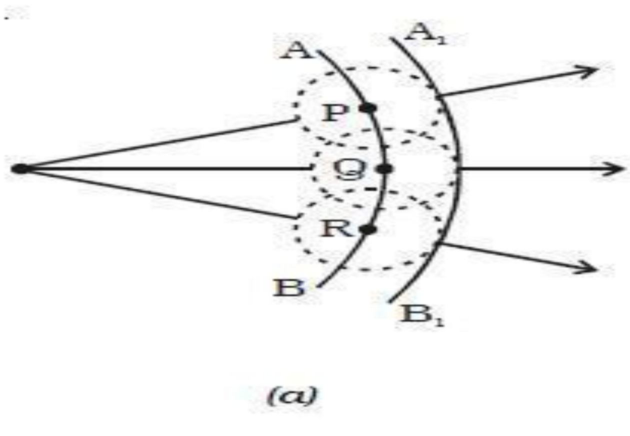
(ii) the new wavefront is the forward envelope of the secondary wavelets at that instant.

Huygens' construction for a spherical and plane wavefront:

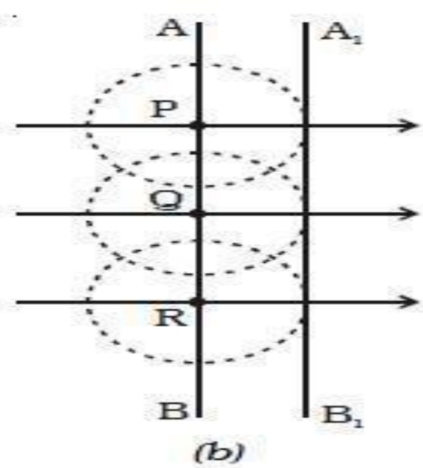
Huygens' construction for a spherical and plane wavefront is shown in Fig. a Let AB represent a given wavefront at a time $t = 0$. According to Huygens' principle, every point on AB acts as a source of secondary wavelets which travel with the speed of light c. To find the position of the wave front after a time t, circles are drawn with points P, Q, R ... etc as centres on AB and radii equal to ct.

These are the traces of secondary wavelets. The arc A1B1 drawn as a forward envelope of the small circles is the new wavefront at that instant.

If the source of light is at a large distance, we obtain a plane wave front A1 B1 as shown in Fig b.



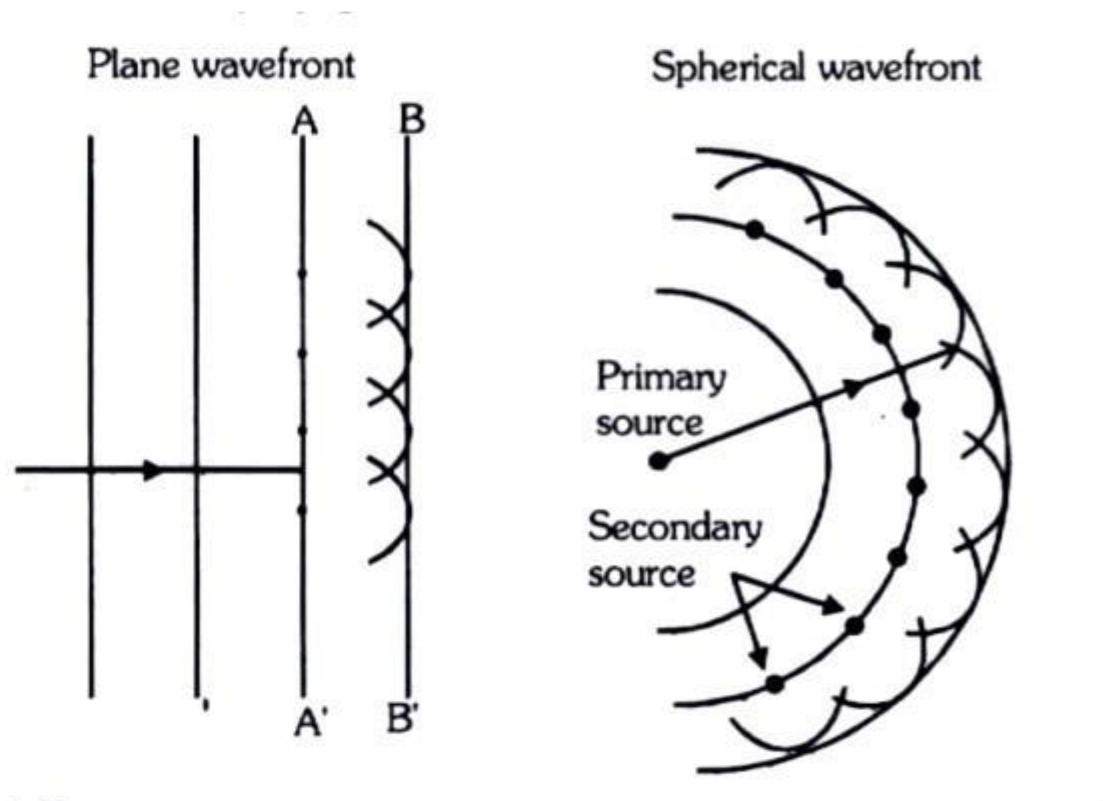
(a)



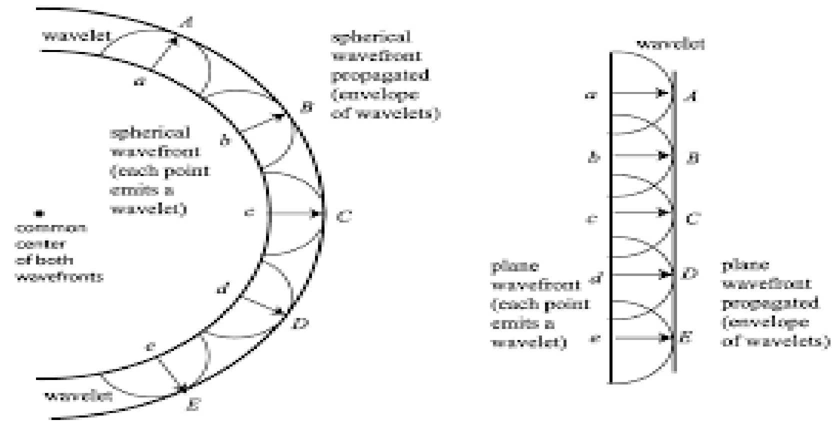
(b)

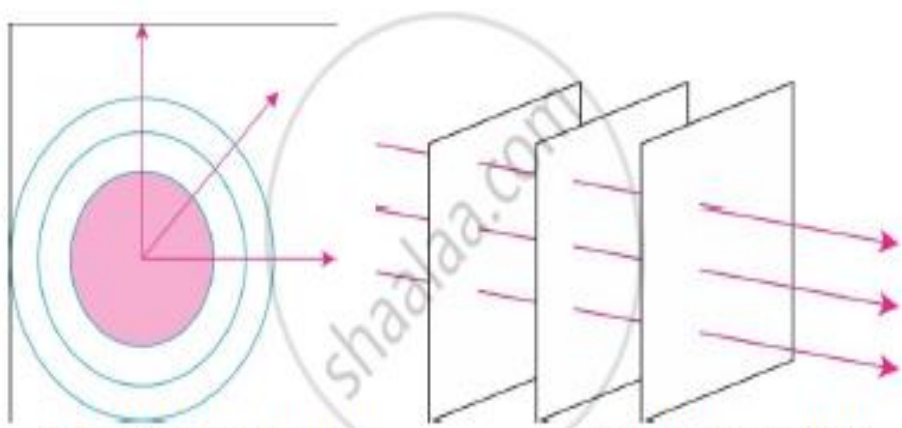
Wave Propagation

Each point (on a wave front) acts as a new source of (spherically, in 3D) propagating wavelets:



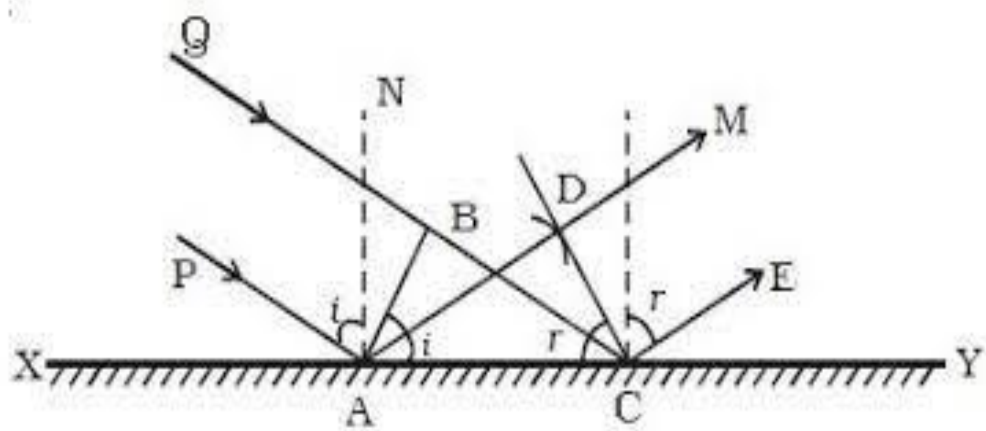
But,we will ignore the backward propagating parts of the wavelets (the reasons are very complicated), and only consider the forward-going (with respect to the previous wave front motion) part of the wave.


wavelet
A
a
spherical wavefront propagated (envelope of wavelets)
B
spherical wavefront (each point emits a wavelet)
b
common center of both wavefronts
c
C
d
D
e
wavelet
E
wavelet
a
A
b
B
c
C
plane wavefront (each point emits a wavelet)
d
D
plane wavefront propagated (envelope of wavelets)
e
E


Spherical wavefront
Plane wavefront

Reflection of a plane wave front at a plane surface

Let XY be a plane reflecting surface and AB be a plane wavefront incident on the surface at A. PA and QBC are perpendiculars drawn to AB at A and B respectively. Hence they represent incident rays. AN is the normal drawn to the surface. The wave front and the surface are perpendicular to the plane of the paper (Fig.). According to Huygens' principle each point on the wavefront acts as the source of secondary wavelet. By the time, the secondary wavelets from B travel a distance BC, the secondary wavelets from A on the reflecting surface would travel the same distance BC after reflection.



Taking A as centre and BC as radius an arc is drawn. From C a tangent CD is drawn to this arc. This tangent CD not only envelopes the wavelets from C and A but also the wavelets from all the points between C and A. Therefore CD is the reflected plane wavefront and AD is the reflected ray.

Laws of reflection

(i) The incident wavefront AB, the reflected wavefront CD and the reflecting surface XY all lie in the same plane.

(ii) Angle of incidence i = ∠ PAN = 900 − ∠ NAB = ∠BAC

Angle of reflection r = ∠ NAD = 900 − ∠ DAC = ∠DCA

$\angle B = \angle D = 90O$

BC = AD and AC is common

∴ The two triangles are congruent

$\angle BAC = \angle DCA$

i.e. $i = r$

Thus the angle of incidence is equal to angle of reflection.

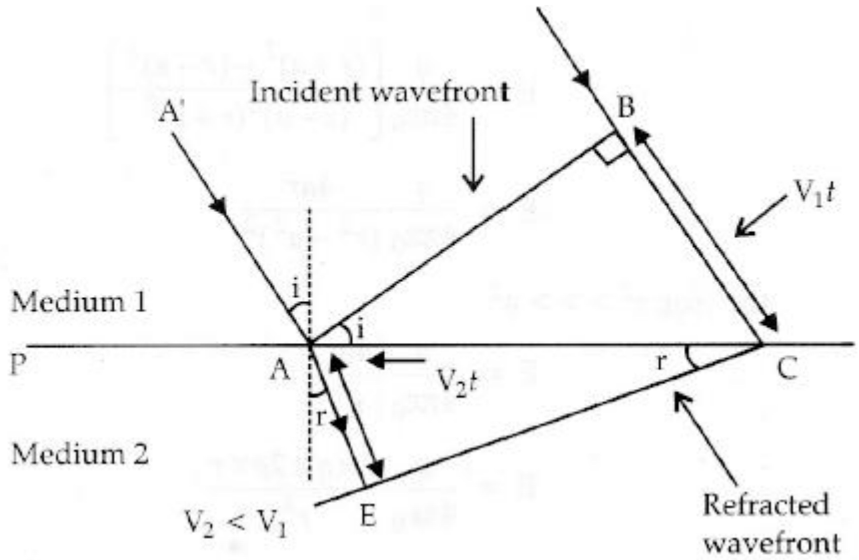
Refraction of a plane wavefront at a plane surface

Let XY be a plane refracting surface separating two media 1 and 2 of refractive indices μ1 and μ2 (Fig). The velocities of light in these two media are respectively v1 and v2. Consider a plane wave front AB incident on the refracting surface at A. PA and QBC are perpendiculars drawn to AB at A and B respectively. Hence they represent incident rays. NAN1 is the normal drawn to the surface. The wave front and the surface are perpendicular to the plane of the paper.
According to Huygen's principle each point on the wave front act as the source of secondary wavelet. By the time, the secondary wavelets from B, reaches C, the secondary wavelets from the point A would travel a distance AD = v2t, where t is the time taken by the wavelets to travel the distance BC.

∴$BC = C_1 t$ and $AD = C_2 t$

Taking A as centre and C2t as radius an arc is drawn in the second medium. From C a tangent CD is drawn to this arc.

Therefore,CD is the refracted plane wavefront and AD is the refracted ray



$$\text{In } \Delta ABC, \quad \sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}$$

$$\text{and in } \Delta AEC, \sin r = \frac{AE}{AC} = \frac{AE}{AC} = \frac{v_2 t}{AC}$$

$$\frac{\sin i}{\sin r} = \left(\frac{v_1 t / AC}{v_2 t / AC}\right)$$

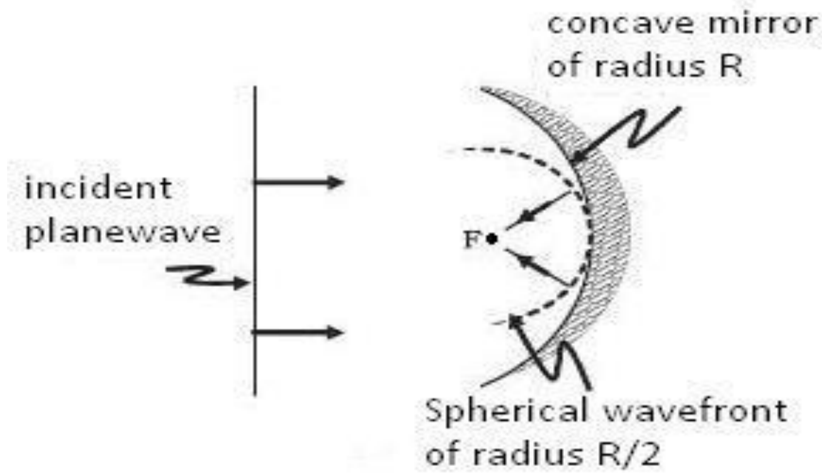
$$= \frac{v_1}{v_2}$$

Constant n21 in above equation is known as refractive index of medium 2 with respect to medium also represented as 1μ2 ,This is Snell's law of refraction.

Further, if λ1 and λ2 denote the wavelengths of light in medium 1 and medium 2, respectively and if the distance BC is equal to λ_1 then the distance AE will be equal to λ_2 (because if the crest from B has reached C in time τ, then the crest from A should have also reached *E* in time τ); thus $\lambda_1 / \lambda_2 = BC/AE = v_1/v_2$
The above equation implies that when a wave gets refracted into a denser medium
($v_1 > v_2$) the wavelength and the speed of propagation decrease but the *frequency* f (= *v*/λ) *remains the same*.

(c) Reflection of a plane wave by a concave mirror

a plane wave is incident on a concave mirror and on reflection we have a spherical wave converging to the focal point F.

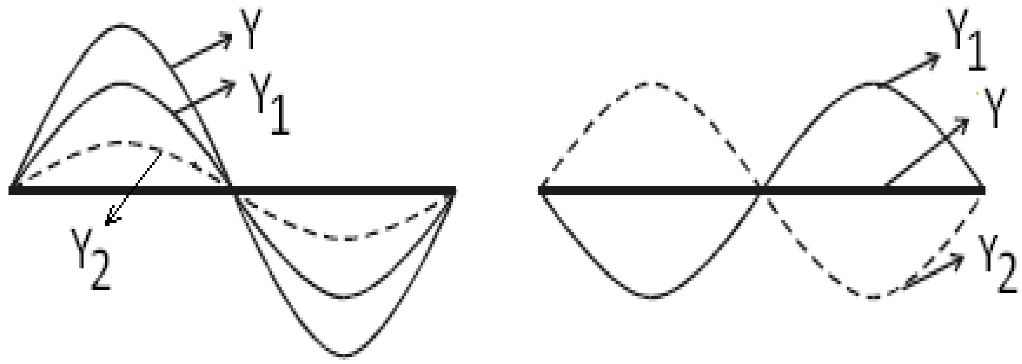


Coherent and incoherent sources

Two sources are said to be coherent if they emit light waves of the same wave length and start with same phase or have a constant phase difference. Two independent monochromatic sources, emit waves of same wave length. But the waves are not in phase. So they are not coherent. This is because, atoms cannot emit light waves in same phase and these sources are said to be incoherent sources.

Superposition principle

When two or more waves simultaneously pass through the same medium, each wave acts on every particle of the medium, as if the other waves are not present. The resultant displacement of any particle is the vector addition of the displacements due to the individual waves. This is known as principle of superposition. If **Y1** and **Y2** represent the individual displacement then the resultant displacement is given by **Y** = **Y1** + **Y2**



Thus, superposition principle describes a situation when more than one waves superpose (i.e. interfere) at a point.

"The effect produced by superposition of two or more wave is called interference".

END