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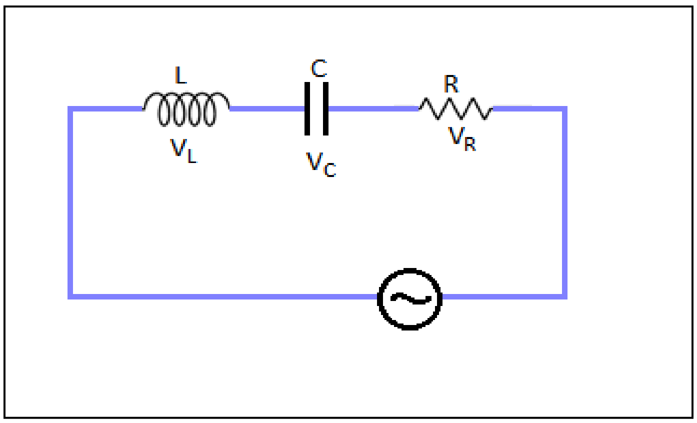
<u>C3T (Electricity and Magnetism)</u> , Topic :- Electrical Circuits

❖ **Sinusoidal Voltage Applied to a Series L-C-R Circuit:** Let us consider an alternating emf. $E(t) = E_0 e^{j\omega t}$ is applied in a circuit containing a resistance R, an inductance L and a capacitance C in series.



Let q(t) be the charge through the capacitor at any instant t and instantaneous currnet is i , where $i = \frac{dq}{dt}$. So the instantaneous voltage,

Across the resistance = Ri

Across the inductance = $-\frac{di}{dt}$

Across the capacitance = $q/_C$

Thus KVL equation for the circuit,

$$Ri + q/_C = E(t) - L\frac{di}{dt}$$

$$\Rightarrow \boxed{Ri + L\frac{di}{dt} + \frac{1}{c}\int i dt = E_0 e^{j\omega t}} \qquad \ldots..(1)$$

➢ Solution:- Let $i(t) = Ae^{j\omega t}$ be the trial solution. From the equation (1)

$$RAe^{jwt} + LAj\omega e^{j\omega t} + \frac{A}{i\omega c} e^{j\omega t} = E_0 e^{i\omega t}$$

$$\Rightarrow A\left(R + j\omega L + \frac{1}{j\omega c}\right) = E_0$$

$$\boxed{\Rightarrow\ A = \frac{E_0}{R + j(\omega L - \frac{1}{\omega c})}} \qquad \ldots\ldots.(2)$$

Thus the current in the circuit,

$$i(t) = \frac{E_0 e^{j\omega t}}{R + j(\omega L - \frac{1}{\omega c})}$$

$$= \frac{R - j(\omega L - \frac{1}{\omega c})}{R^2 + (\omega L - \frac{1}{\omega c})^2} . E_0 e^{j\omega t}$$

$$= \frac{E_0}{\sqrt{(R^2 + (\omega L - \frac{1}{\omega c})^2}} \left[\frac{R}{\sqrt{(R^2 + (\omega L - \frac{1}{\omega c})^2}} - j \frac{(\omega L - \frac{1}{\omega c})}{\sqrt{(R^2 + (\omega L - \frac{1}{\omega c})^2}} \right] e^{j\omega t}$$

Put, $\frac{R}{\sqrt{(R^2 + (\omega L - \frac{1}{\omega c})^2}} = cos\theta$

$$\frac{(\omega L - \frac{1}{\omega c})}{\sqrt{(R^2 + (\omega L - \frac{1}{\omega c})^2}} = sin\theta$$

$$.: tan\theta = \frac{\omega L - \frac{1}{\omega c}}{R}$$

$$\therefore i = \frac{E_0}{\sqrt{(R^2+(\omega L-\frac{1}{\omega c})^2}} e^{-j\theta} . e^{j\omega t}$$

$$\Rightarrow \boxed{\boldsymbol{i = i_0 e^{j(\omega t-\theta)}}} \qquad \text{Where } i_0 = \frac{E_0}{\sqrt{(R^2+(\omega L-\frac{1}{\omega c})^2}}$$

$$i = i_0[\cos(\omega t-\theta) + j sin(\omega t-\theta)]$$

$$i_0 \cos(\omega t-\theta) \Rightarrow Real\ part\ of\ current$$
$$i_0 sin(\omega t-\theta) \Rightarrow Imaginary\ part\ of\ current$$

- **Peak Value of Current:**

$$i_0 = \frac{E}{|Z|} = \frac{E_0}{\sqrt{(R^2+(\omega L-\frac{1}{\omega c})^2}}$$

- **Impedance:** $z = \frac{E}{i} = R + j(\omega L - \frac{1}{\omega c})$

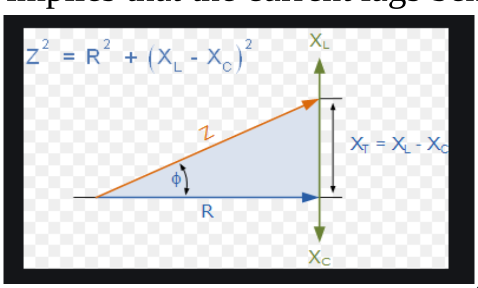
$$\vec{Z} = \vec{R} + j(\omega\vec{L} - \frac{1}{\omega c})$$

$$|Z| = \sqrt{(R^2 + (\omega L - \frac{1}{\omega c})^2}$$

Inductive reactance $X_L = \omega L$

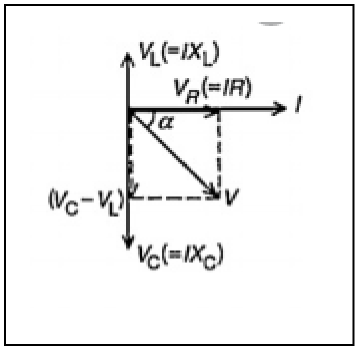
Capacitive reactance $X_c = \frac{1}{\omega c}$

- Impedance diagram and phasor diagram:-
  a) When $\omega L > \frac{1}{\omega c}$ i.e. inductive reactance is greater than capacitive reactance. Here $\emptyset$ is positive; which implies that the current lags behind the emf by an angle $\emptyset$



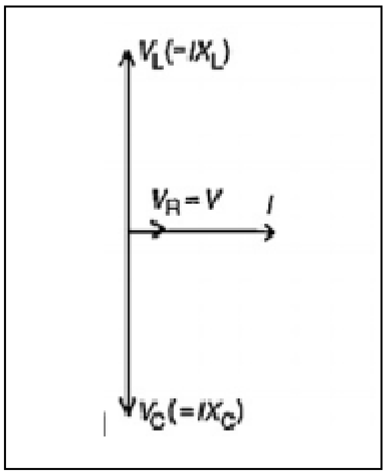
, where $\emptyset = \tan^{-1}\frac{\omega L-\frac{1}{\omega c}}{R}$.

b) When $\frac{1}{\omega c} > \omega L$ i.e. when capacitive reactance is greater than inductive reactance.



Here $\alpha = -Ve$; which implies that the current leads the applied emf by an angle $\alpha = \tan^{-1}(\frac{\frac{1}{\omega c}-\omega L}{R})$

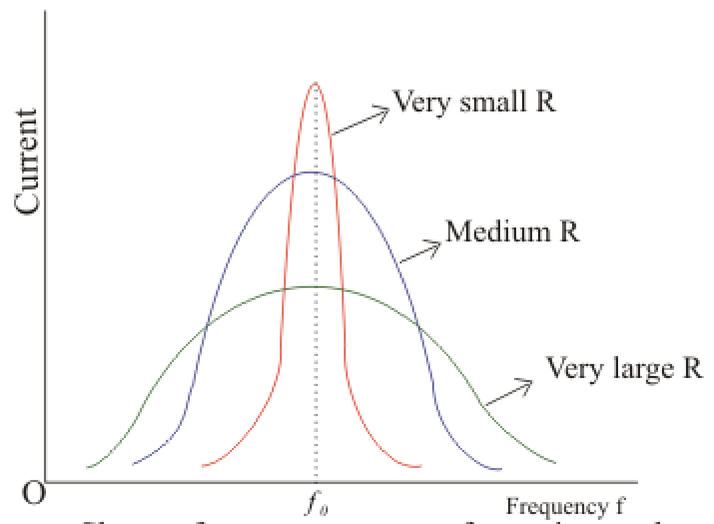
c) When $\omega L = \frac{1}{\omega c}$ i.e when inductive reactance is equal to capacitive reactance.



Here $\theta = 0$ i.e the circuit is purely resistive.

- **Series Resonant Circuit:** The circuit containing inductor(L), capacitor(c) and resistor(R) are connected in series and subjected to an alternating emf. Where
  (i) The circuit behaves as purely resistive.
  (ii) Impedance is minimum.
  (iii) Current is maximum
  (iv) The emf and current will be in same phase.
  Then the circuit is called series resonant circuit.

- **Sharpness of Resonance of a Series L-C-R Circuit:** For a series L-C-R circuit the maximum current $i_0 = \frac{E_0}{\sqrt{(R^2+(\omega L-\frac{1}{\omega c})^2}}$; At resonance , $\omega L = \frac{1}{\omega c}$ then $i_0 = \frac{E_0}{R}$



Shape of resonance curves for various values of R

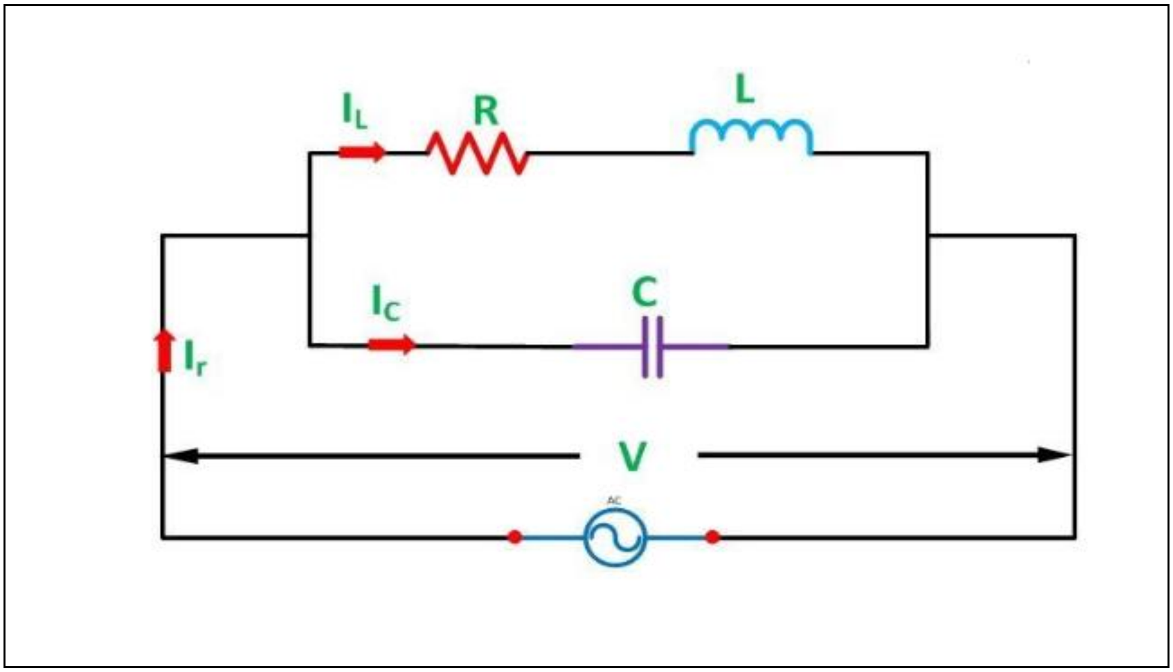
The curves plotting the current verses frequency (f and ω) are known as resonance curves. From the figure we see that when the resistance in the circuit is reduced the resonance curve become sharper. The peak value of i shows that, the circuit responds only to the frequency exactly equal to the natural frequency of the circuit $\boldsymbol{\omega_0 = \frac{1}{\sqrt{Lc}}}$ **.** This resonance is sharp. The sharpness of resonance is a measure of the rate of fall of amplitude from its maximum value at resonance frequency on either side of it.

- Quality Factor: Sharpness of resonance curve is determined by quality factor, called "Q" of the circuit. Which is defined as the ratio of the reactance $(X_L or X_C)$ to the impedance at resonant frequency is called Quality factor of the circuit. i.e.

$$Q = \left.\frac{X_L}{Z}\right|_{\omega_0} = \frac{\omega_0 L}{R}$$

$$\text{Or, } Q = \left.\frac{X_c}{Z}\right|_{\omega_0} = \frac{1}{\omega_0 c R}$$

❖ **Parallel L-C-R Circuit:** Let the combination is connected to alternating emf, $E = E_0 e^{j\omega t}$. If Z be the total impedance for the combination then,



$$\frac{1}{Z} = \frac{1}{Z_{L-R}} + \frac{1}{Z_c}$$

$$\Rightarrow \frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega c$$

$$\Rightarrow z = \frac{R + j\omega L}{(1 - \omega^2 LC) + j(\omega cR)}$$

$$= \frac{R + j\omega L}{(1 - \omega^2 LC) + j(\omega cR)} \times \frac{(1 - \omega^2 LC) - j(\omega cR)}{(1 - \omega^2 LC) - j(\omega cR)}$$

$$.: z = \frac{R + j[\omega L - \omega c(\omega^2 L^2 + R^2)]}{(1 - \omega^2 LC)^2 + \omega^2 c^2 R^2}$$

$$= \frac{R}{(1-\omega^2 LC)^2 + \omega^2 c^2 R^2} + j\omega \frac{L - c(\omega^2 L^2 + R^2)}{(1-\omega^2 LC)^2 + \omega^2 c^2 R^2}$$

$$= R_0 + j\omega L_o \text{ (say)}$$

Where $R_0 = \frac{R}{(1-\omega^2 LC)^2+\omega^2 c^2 R^2}$; effective resistance of the coil.

$L_o = \frac{L-c(\omega^2 L^2+R^2)}{(1-\omega^2 LC)^2+\omega^2 c^2 R^2}$; effective inductance of the coil.

- **Magnitude of Impedance:-**

$$|z| = \sqrt{{R_0}^2 + \omega^2 {L_0}^2}$$

$$|z| = \left[\frac{R^2 + \omega^2 L^2}{(1-\omega^2 LC)^2 + \omega^2 c^2 R^2}\right]^{1/2}$$

- **Resonant Frequency :-** The parallel at which the circuit is purely resistive or current will be in same phase with the emf is known as resonant frequency. At $\omega = \omega_0$, $L_0 = 0$

$$\Rightarrow L - c({\omega_0}^2 L^2 + R^2) = 0$$

$$\Rightarrow {\omega_0}^2 L^2 + R^2 = 0$$

$$\Rightarrow {\omega_0}^2 L^2 = \frac{L}{c} - R^2$$

$$\Rightarrow {\omega_0}^2 = \frac{1}{Lc} - \frac{R^2}{L^2}$$

$$\boxed{\Rightarrow \omega_0 = \sqrt{\frac{1}{Lc} - \frac{R^2}{L^2}}}$$

Parallel resonant frequency, $$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{Lc} - \frac{R^2}{L^2}}$$

- **Condition of Parallel Resonance:** For parallel resonance , ω_0= real (must)

$$\Rightarrow \sqrt{\frac{1}{Lc} - \frac{R^2}{L^2}} > 0$$

$$\Rightarrow \frac{1}{Lc} - \frac{R^2}{L^2} > 0$$

$$\Rightarrow R^2 < \frac{L}{c}$$

$$\Rightarrow R < \sqrt{\frac{L}{c}}$$

Hence for parallel resonance resistance R should be kept as low as possible.