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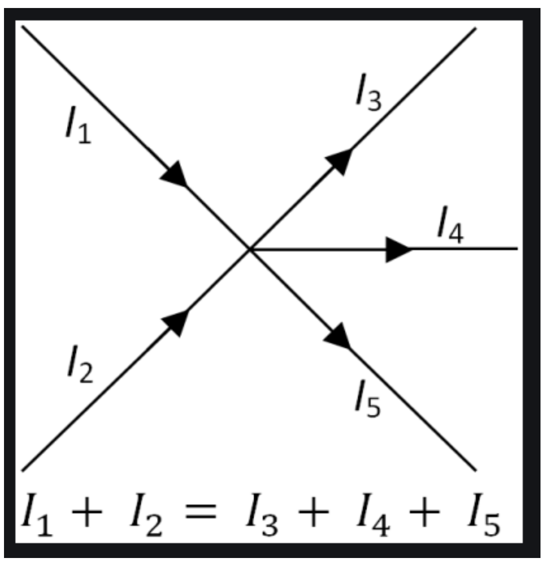
<u>C3T (Electricity and Magnetism)</u> , Topic :- Electrical Circuits

- ❖ **Kirchhoff's Laws:** Kirchhoff's offered two laws which are applicable to complicated networks of conductor.
- ➢ **Kirchhoff's 1st Law:** The algebraic sum of currents meeting at a junction point of conductors is always zero.



- **Explanation:** Let $i_1, i_2, i_3 \ldots\ldots$ be the currents through conductors connected at a junction point. Then

$$i_1 + i_2 - i_3 + i_4 - i_5 = 0$$

i.e. $$\boxed{\sum \boldsymbol{i} = \mathbf{0}}$$

- Sign Convension: The current towards the junction point = + Ve

  The current outwards the junction point = -Ve

- **Proof :** From the equation of continuity,

$$\vec{\nabla}.\vec{j} = -\frac{\partial \rho}{\partial t}$$

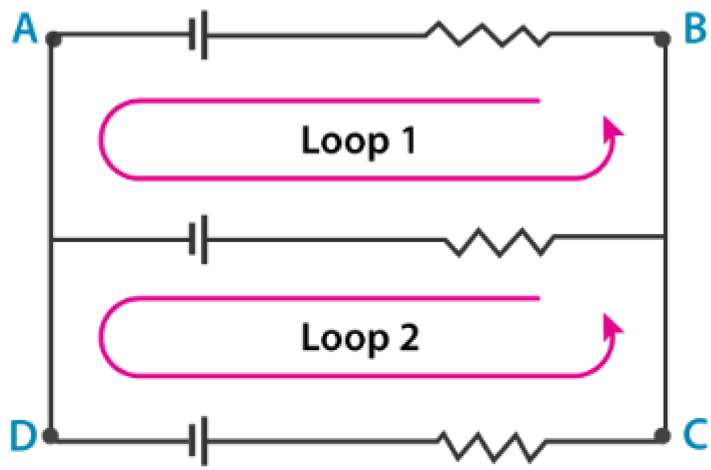
$$\Rightarrow \iiint \vec{\nabla}.\vec{j}\,dv = -\iiint \frac{\partial \rho}{\partial t}\,dv$$

$$\Rightarrow \oiint \vec{j}.\hat{n}ds = -\iiint \frac{\partial \rho}{\partial t}\,dv$$

$$\Rightarrow \quad \boxed{\sum \boldsymbol{i} = \mathbf{0}} \quad ;\text{ for steady current } \frac{\partial \rho}{\partial t} = 0$$

N.B- 1. Kirchhoff's 1st law being the law of conservation of electric charge.
2. It is known as Kirchhoff's current law or KCL

- **Kirchhoff's 2nd Law:** The algebraic sum of the product of current and resistance in each branch of a closed network of conductance is equal to the total emf in the circuit.

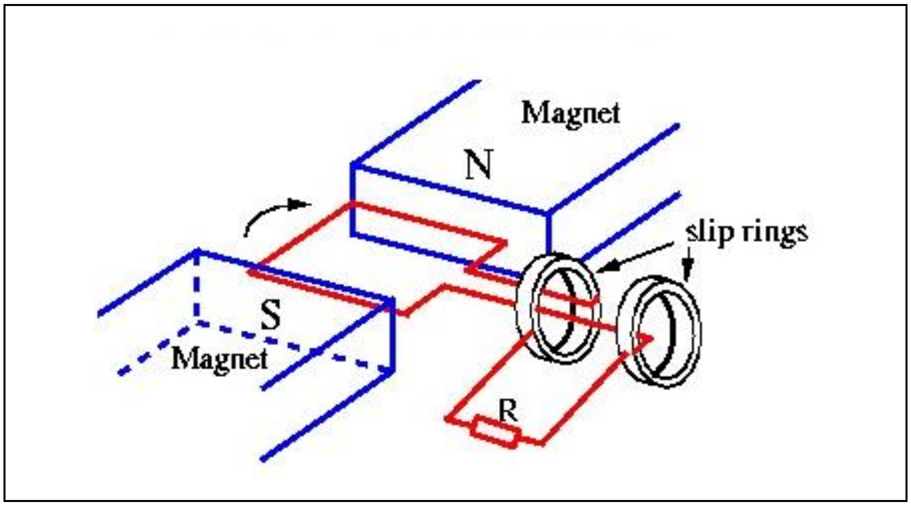


- Explanation : Let us take a closed loop ABCDEA, consisting branches having resistances $r_1, r_2, r_3, and\ e_1, e_2, e_3$. Then

$$\boxed{\sum \boldsymbol{ir} = \sum \boldsymbol{e}}$$

N.B – 1. 2nd law being the law of conservation of energy.
2. It is called Kirchhoff's voltage law or KVL.

❖ **Production of Alternating Current:** Let at any moment the magnetic flux linked up with the coil,



$$\varphi = N\int \vec{B}.\hat{n}ds$$

Where N is the total no. of turns of the coil, B is the magnetic flux density, $\hat{n}$ is the unit vector normal to the plane of the coil.

If θ be the angle between $\vec{B}$ and $\hat{n}$, then

$$\varphi = BANcos\theta \qquad \ldots.(1)$$

If ω be the angular velocity then $\theta = \omega t$

$$\therefore \text{Induced emf } e = -\frac{d\emptyset}{dt}$$

$$= -\frac{d}{dt}(BANcos\omega t)$$

$$= BAN\omega sin\omega t \qquad \ldots(2)$$

Where $BAN\omega = E_0$, the peak value of induced emf.

Thus the current passing through R,

$$i = \frac{E}{R} = \frac{E_0}{R}sin\omega t$$

$$\Rightarrow i = i_0 sin\omega t \qquad .....(3)$$

Where $i_0 = \frac{E_0}{R}$ the peak value of current.

❖ **Average Value of Alternating emf / current over Complete Cycle :** The instantaneous value of alternating emf

$$\boldsymbol{E = E_0} sin\omega t \qquad ....(1) \text{ Taking imaginary part}$$

Thus average value of alternating emf. over complete cycle ,

$$E_{av} = \frac{1}{T}\int_0^T E dt$$
$$= \frac{1}{T}\int_0^T E_0 sin\omega t dt$$
$$= -\frac{E_0}{\omega T}[cos\omega t]^{T}{}_{0}$$
$$= -\frac{E_0}{2\pi}[cos2\pi - cos0] \qquad \omega = \frac{2\pi}{T}$$
$$= 0$$

Similarly, $i_{av} = 0$

❖ **Average Value of Alternating emf / current over Half Cycle :**

$$E_{av}|_{T/2} = \frac{1}{T/2}\int_0^{T/2} E dt$$
$$= \frac{2}{T}\int_0^{T/2} E_0 sin\omega t dt$$
$$= -\frac{iE_0}{wT}\left[cos\frac{wT}{2} - cos0\right]$$
$$= \frac{2E_0}{\pi}$$

Similarly, $i_{av}|_{T/2} = \frac{2i_0}{\pi}$

❖ **Root mean square (R.M.S) value of Alternating emf/current:-** The mean square value of alternating emf over a full cycle,

$$E_{m.s} = \frac{1}{T}\int_0^T E^2 dt$$

$$= \frac{1}{T}\int_0^T E^2 sin^2\omega t dt$$

$$= \frac{{E_0}^2}{2T}\int_0^T (1 - cos2\omega t)dt$$

$$= \frac{{E_0}^2}{2}$$

Thus root mean square (r.m.s) value of alternating emf,

$$\boldsymbol{E_{r.m.s} = \frac{E_0}{\sqrt{2}}}$$

Similarly, $\boldsymbol{i_{r.m.s} = \frac{i_0}{\sqrt{2}}}$

- **Form Factor :** The ratio of r.m.s value of alternating emf. to the average value (over half cycle) is known as form factor.

  .: Form factor F$= \frac{E_{r.m.s}}{E_{av}} = \frac{\pi}{2\sqrt{2}} = 1.11$

$$\boldsymbol{F = 1.11}$$

- **Average Power calculation:** Average power for a complete cycle of alternating currnt , average power,

$$P_{av} = \frac{1}{T}\int_0^T E_0 i_o sin^2\omega t dt = \frac{E_0 i_o}{T} \times \frac{T}{2}$$

$$= \frac{E_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}}$$

$$\boldsymbol{.:\ P_{av} = E_{r.m.s} \times i_{r.ms}}$$