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DSE4T (Experimental Techniques)

Topic – Signals and Systems (Part – 2)

We have already discussed part 1 of this e-report.

Now let us continue part 2 of it.

Random Signals and Noise:

In electrical engineering one often encounters signals that do not have a precise mathematical description, since they develop as random functions of time. Sometimes this random development is caused by a single random variable, but often it is a consequence of many random variables. In other cases the causes of randomness are not clear and a description is not possible, but the signal is characterized by means of measurements only. A random time function may be a desired signal, such as an audio or video signal or it may be an unwanted signal that is unintentionally added to a desired information or signal and disturbs the desired signal. We call the desired signal a random signal and the unwanted signal as *noise*. However, the latter often does not behave like noise in the classical sense, but it is more like interference. Then it is information bearing signal as well, but undesired. A desired signal and noise (or interference) can in general, not be distinguished completely, by means of well-defined signal processing in a receiver, the desired signal may be favoured in a maximal way whereas the disturbance is suppressed as much as possible. In all cases a description of the signals is required in order to be able to analyse its impact on the performance of the system under consideration. Especially in communication theory this situation often occurs. The random character as a function of time makes the signals difficult to describe and the same holds for signal processing or filtering. Nevertheless, there is a need to characterize these signals by a few deterministic parameters that enable the system user to assess the performance of the system.

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Fluctuations and Noise in Measurement Systems:

When a phenomenon is observed and recorded, information about the process involved is obtained from the value of a quantity, e.g. light intensity or number of counts per second. It is desirable that this value reflect the stable or true behaviour of the phenomenon. However, the recorded quantity usually shows *variations* with time as may be observed from a plot of the data shown in Fig. 1.

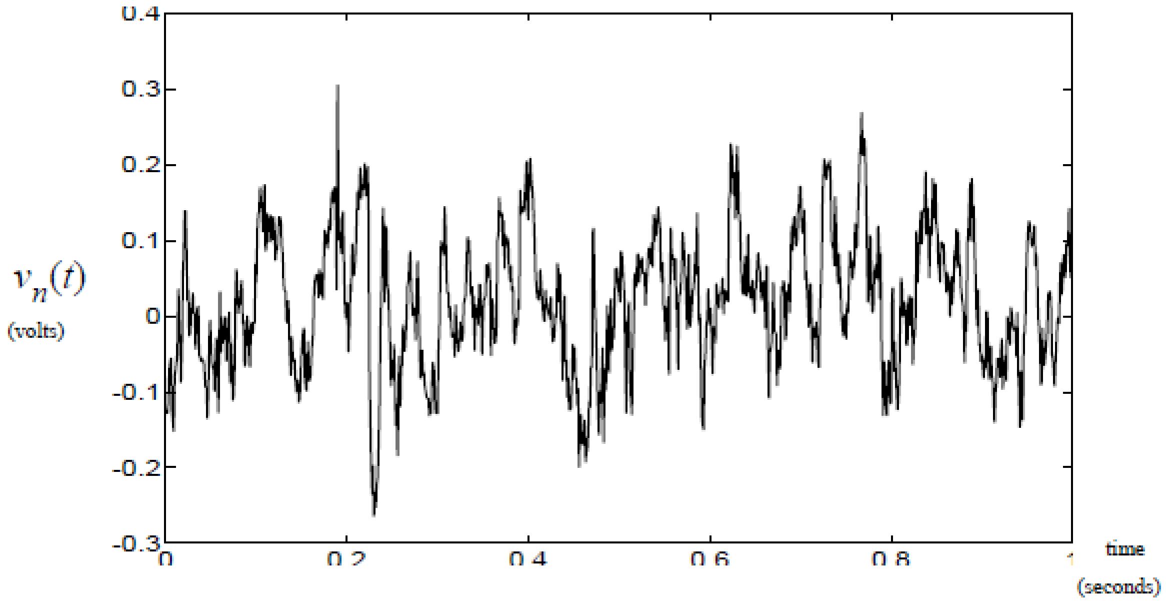

Fig. 1

The origin of these fluctuations may be two-fold, for example

(a) Inherent in the process, e.g. the flickering of a candle flame or

(b) Measurement-related, e.g. due to instabilities in the amplification of a measuring instrument.

In instrumentation, these fluctuations are the noise, we introduced before. If the noise obeys Poisson statistics, the width of the distribution of measured voltages is given by the square root of the mean value.

Signal to Noise Ratio. The relative magnitude of the noise is defined by a parameter known as signal to noise ratio (in short, SNR). It is defined by the ratio between the average value of the signal to the RMS value of the fluctuation. Mathematically,

$$SNR = \frac{S}{N}$$

where S is the mean signal value and N is the fluctuation value. Alternatively, it can also be defined in terms of a power ratio (in units of dB). If P is the mean signal power and S_P is the noise power, then SNR can be defined as

$$SNR \text{ (in dB)} = 10 \log\left(\frac{P}{S_P}\right)$$

The purpose of many measurement techniques is to increase the signal to noise ratio so that the detecting capability of the signal is increased. Simple amplification cannot do this since the noise is amplified by the same factor as the signal. Means have to be sought to distinguish the signal from the noise.

Noise Figure. Noise may be contributed by each stage of a measurement system, e.g. the initial phenomenon, the detecting instrument, the amplifier chain and the display. The noise contributed by each stage is defined by the corresponding noise figure (F) which is expressed in terms of the ratio of the SNR at the input to the SNR at the output. It means

$$F = \frac{SNR(input)}{SNR(output)}.$$

If no noise is contributed by the stage then we have $F = 1$, also $F > 1$ for all real systems and $F = 1.3$ for many semiconductor devices.

Noise in Frequency Domain:

The $v_n - t$ graph in Fig. 1 shows the time variation of a quantity, called a *time series data*. The average (signal) value is constant (or slowly varying) with time (i.e. DC) while the noise has a complex waveform. These differences are most conveniently described by the frequency spectrum, for which Fourier Analysis becomes very crucial.

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The Fourier Transform converts an amplitude variation as a function of time $f(t)$ to a complex amplitude and phase spectrum as a function of frequency $F(\omega)$ in the following manner

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$

The amplitude $|F(\omega)|$ has units of the signal unit (e.g. V or A) per Hz and ω is the angular frequency. The Fourier power spectrum is calibrated by noting that, in principle, the power spectrum S_P is given by the square of the signal amplitude (e.g. voltage amplitude) v_n generated across a standard resistance ($S_P \propto {v_n}^2$).

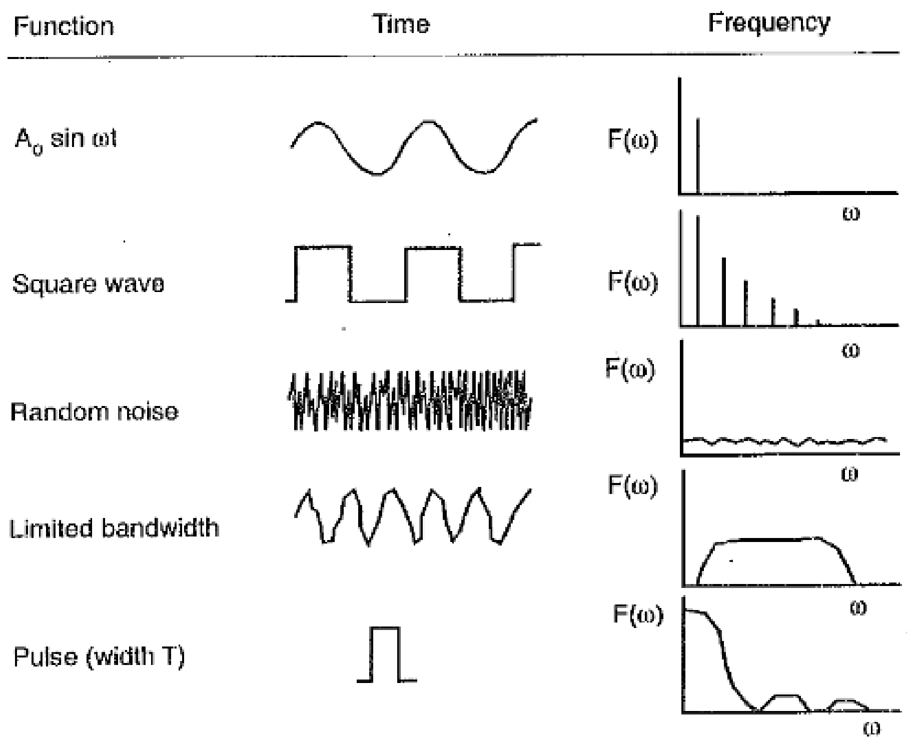

Fig. 2

Important technical examples are shown in Fig. 2. For a completely random waveform, an infinite range of frequencies exists. For a pulsed waveform, if the pulse with duration T is infinitely short ($T \to 0$), $F(\omega)$ extends to infinite frequency. Instrumental considerations invariably preclude both of these possibilities.

Frequency Bandwidth. The importance of the frequency domain is that it can describe the frequency bandwidth in which the signal information is carried and distinguish this from the frequency bandwidth over which noise exists. For example, the waveform shown in Fig. 1 can be analyzed in the frequency domain as having two components, namely one defining the signal and other defining the noise. This is shown in Fig. 3.

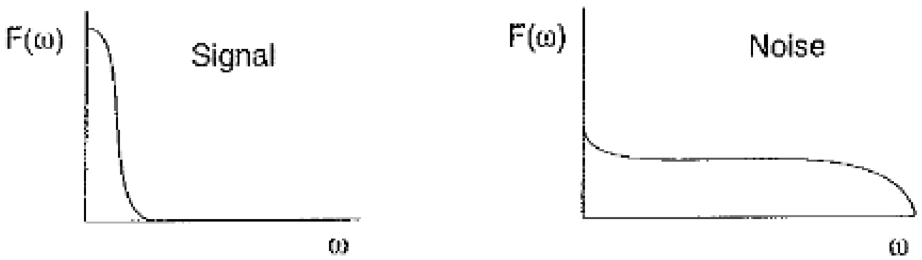

Fig. 3

Because the signal is likely to be well defined in time, the frequency spectrum which contains the signal information is relatively limited in width. The noise spectrum originates from a wide variety of sources and is much broader in frequency bandwidth. The frequency limiting effects of various stages in the measurement system must also be described. Instruments (amplifiers) are built with a specific frequency bandwidth Δf. Circuit components form low pass and high pass filters. All frequency components of a signal falling within the measurement bandwidth will be detected, all those outside will not be observed. Signal to noise ratios must therefore be calculated taking into account the frequency bandwidth of the measurement system. The frequency bandwidth for a measurement system may be represented by a frequency dependent transfer function $H(\omega)$. In the ideal case, the frequency response is limited at a specific frequency (shown in Fig. 4(a)).

For analog circuits such a response cannot be achieved, although it is possible using digital techniques. The transfer function for real analog systems falls off more slowly (as shown in Fig. 4(b)). For such a low-pass $R-C$ circuit, the cut-off frequency f_C is defined as the frequency at which transfer function $H(\omega)$ falls to $\frac{1}{\sqrt{2}}$ of its initial value (equivalently by $\approx$ 3 dB). The frequency-axis for

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graphs of the type given in Fig. 4 is usually plotted on a logarithmic scale. This can be misleading with respect to the range of frequencies over which the response is frequency dependent. A linear plot of the data shown in Fig. 4(b) will clearly demonstrate that the rate of fall-off of circuit response with frequency for analog $R - C$ circuits is relatively slow.

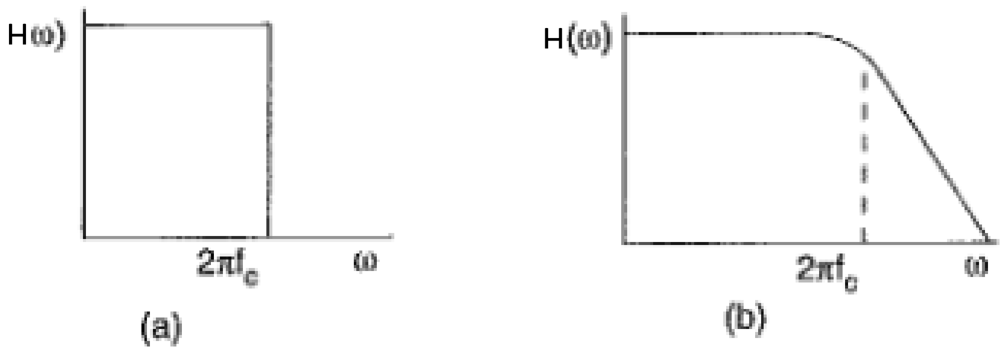

Fig. 4

Sources of Noise:

Some experimental systems are inherently unstable and lead to measurements which fluctuate in time. Examples are optical fluctuations in starlight passing through the earth's atmosphere, radiowaves in the ionosphere or bad electrical connections at contacts to a semiconductor. In general, this inherent noise has to be treated statistically or by noting the variation over periods of time. It is always important to treat instabilities with interest, observations of fluctuations in starlight led to the discovery of pulsars and quasars, while consideration of unstable microwave emissions from metal junctions at low temperatures led Brian Josephson of Cambridge University to win a Nobel Prize for the famous Josephson Effect. Some sources of noise are inherent to particular physical systems. These include Thermal Noise in resistors and Shot Noise in systems where charge collection occurs.

Thermal Noise or Johnson Noise. It occurs due to the thermodynamic fluctuations of the electron gas in a conductor. The density of the electron gas in a resistance R fluctuates spatially leading to variations in potential difference v_n between the contacts. The variations arise because of the thermal motion of the

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electron gas and the possible frequencies are, in principle, unlimited. This is termed as *white noise*. The magnitude of Johnson noise may be estimated by considering the energy which is available to generate the fluctuations and the manner whereby the power is communicated to an external circuit, as depicted in Fig. 5 .

Thermal energy available at temperature T is $\sim k_B T$ (where k_B is Boltzmann's constant). This energy is spread across the entire frequency spectrum. If the measurement system has a limited bandwidth Δf, only a fraction of the energy will be measured. Noise power available to a measurement system of frequency bandwidth Δf is given by $\sim k_B T \Delta f$. The resistor may be treated as a perfect noiseless resistance R in series with an internal voltage source v_n. The maximum power that can be transferred to an external load resistance R_L occurs when $R_L = R$. The terminal voltage is then $v_0 = \frac{v_n}{2}$.

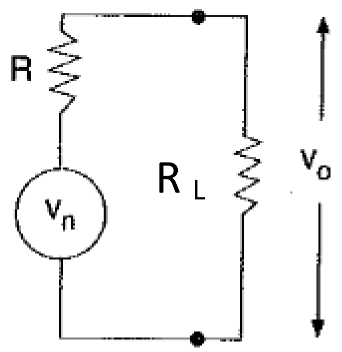

Fig. 5

So, the noise power is given by $k_B T \Delta f = \frac{{v_0}^2}{R_L} = \frac{{v_n}^2}{4R}$, from which the RMS noise voltage is calculated as

$$v_n = \sqrt{4 k_B T R \Delta f}.$$

The frequency spectrum is flat (shown in Fig. 6), in principle to infinite frequency, as discussed before. In practice, it is limited by circuit connections which act as high or low pass filters. Johnson noise exists in all resistive

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circuits, which can't be eliminated, but its effect can be reduced, by simply lowering the temperature of the system.

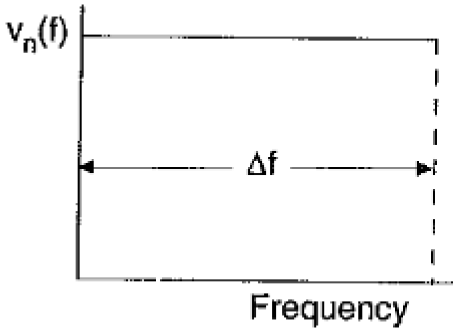

Fig. 6

Shot Noise. It occurs due to the collection of electrons at an electrode or by diversion over a barrier. This type of noise is characteristic of any system in which charge collection occurs statistically. This takes place in the current flowing to the anode of a photomultiplier, in minority carriers flowing across a junction diode or across the base of a junction transistor to the collector. It is a direct consequence of *Poisson Statistics*. As shown in Fig. 7, let us consider N number of charges q collected in time T, so the mean current $I_0 = \frac{Nq}{T}$.

Since it is a random process, there will be a statistical uncertainty of $\pm\sqrt{N}$ in the number of charges collected in this time, $I = I_0 \pm I_{rms}$ where

$$\frac{I_{rms}}{I_0} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}} = \sqrt{\frac{q}{I_0 T}}.$$

The RMS value of the shot noise current is therefore

$$I_{rms} = \sqrt{\frac{qI_0}{T}}.$$

This is a result of a counting process over a time interval T. We can therefore replace it with the equivalent bandwidth of a pulse of this time duration given by $\Delta f = \frac{1}{2T}$. Therefore, RMS shot noise current is given by

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$$I_{rms} = \sqrt{2qI_0\Delta f}.$$

For electrons (carrying a charge of e), the square of the RMS current is given by

$$I_{rms}{}^2 = 2eI_0\Delta f.$$

It is important to note that these fluctuations can occur at any frequency and therefore the shot noise spectrum is white, similar to Johnson noise spectrum.

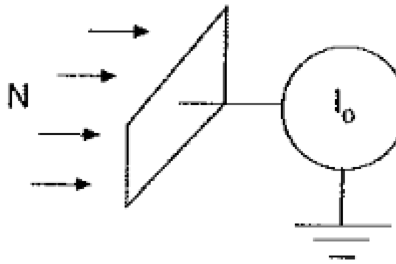

Fig. 7

$\mathbf{\frac{1}{f}}$ **Noise or Flicker Noise.** Many electrical systems show more noise power at low frequencies than is predicted by the thermodynamic formulae. This excess noise is often determined by unknown effects at surfaces, contacts and barriers and improves as devices are better designed or manufactured for a longer period of time. This type of noise is called $\frac{1}{f}$ noise or Flicker Noise. Such a name has been given since the noise power varies as $\frac{1}{f}$ (and therefore the noise signal varies as $\frac{1}{\sqrt{f}}$, shown in Fig. 8). It is also associated with the generation and recombination of minority carriers in semiconductors.

An empirical law known as the Hooge Law describes many noise results in semiconductor materials, given below

$$S_P \propto \frac{1}{f} = \left(\frac{\alpha}{N_f}\right)\frac{\Delta f}{f}$$

where N_f is the number density of electrons or holes, Δf is the measuring bandwidth, f is the frequency and $\alpha \approx 2 \times 10^{-3}$ Wcm^{-3} is an empirical

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constant. This type of noise is not well understood, but it is very prevalent and consideration of its effects is an essential part of instrumentation design. It is clear that a power spectrum of the form $\frac{1}{f}$ cannot extend to zero frequency. An estimate of the cut-off frequency (f_C) below which $\frac{1}{f}$ noise can be ignored is given by $f_C = \frac{1}{2\tau}$, where τ is the time over which the noise is measured.

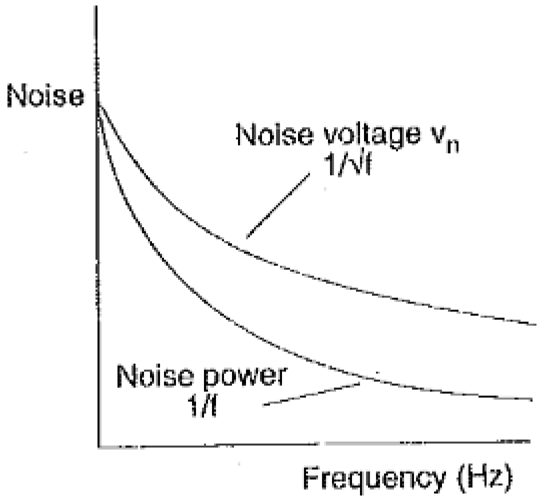

Fig. 8

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(All the figures have been collected from the above mentioned references)

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