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C13T (Electromagnetic Theory)

Topic – EM Wave Propagation in Unbounded Media

(Part – 2)

We have already discussed part 1 of this e-report.

Now let us continue part 2 of it.

Plasma:

Plasma exists in many forms in nature and has a widespread use in science and technology. It is a special kind of partially ionized gas and in general consists of positively charged ions, electrons and neutrals (i.e. atoms, molecules, radicals etc.). Here we call an ionized gas as plasma if it is quasi-neutral and its properties are dominated by electric and/or magnetic forces. Owing to the presence of free charge carriers, plasma reacts to electromagnetic fields, conducts electrical current and possesses a well-defined space potential. Positive ions may be singly charged or multiply charged. For a plasma containing only singly charged ions, the ion population is adequately described by the ion density n_i. Besides the ion density, we characterize a plasma by its electron density n_e and the neutral density n_n.

Quasi-neutrality of a plasma means that the densities of negative and positive charges are (almost) equal. In the case of plasma containing only singly charged ions, this means that

$$n_i \approx n_e.$$

In the presence of multiple charged ions, we have to modify this relation. If z is the charge number of a positive ion and n_z is the density of z-times charged ions, the condition of quasi-neutrality reads

$$n_e \approx \sum_z z\, n_z.$$

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Plasma Oscillations and Plasma Frequency. To investigate quasi-neutrality further, let us assume that a cloud of electrons in plasma has moved to a certain area, forming a negative space charge there (shown in Fig. 1). A similar ion cloud is left without electrons in a distance $L \approx x$, forming a positive space charge. Thus one obtains between these space charge clouds an electric field having its value E at the mutual borders having cross-sectional area A. We can now estimate the value of E by using Poisson's Equation, given by

$$\int \vec{E}.d\vec{S} = EA = \frac{en_e xA}{\varepsilon_0}$$

$$\text{or } E = \frac{en_e x}{\varepsilon_0}$$

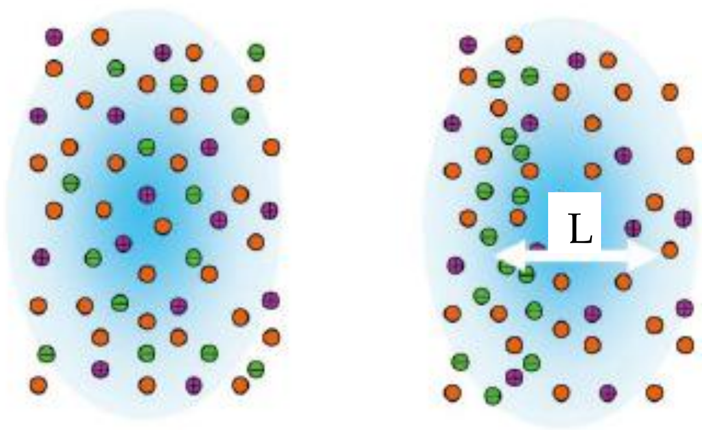

Fig. 1

Thus we obtain for the movement of, say, electrons, under the action of the restoring force given by $F = -eE = -\frac{e^2 n_e x}{\varepsilon_0}$.

The equation of motion of the electrons thus becomes

$$m_e \frac{d^2x}{dt^2} = F = -\frac{e^2 n_e x}{\varepsilon_0}$$

$$\text{or } \frac{d^2x}{dt^2} + \frac{n_e e^2 x}{\varepsilon_0 m_e} = 0$$

$$\text{or } \frac{d^2x}{dt^2} + {\omega_p}^2 x = 0.$$

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This is the equation of a simple harmonic motion with natural frequency ω_p, known as the *plasma frequency* (or *electron plasma frequency*), where

$$\omega_p = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}.$$

These *plasma oscillations* are oscillations of the electron charge cloud as a whole. The inert ions are considered to remain at rest.

Electrical Conductivity of Plasma. As a first step we consider EM waves in plasma without an external magnetic field, that is, for $\vec{B} = 0$. In contrast to vacuum, we may have space charge and electric current in plasma. So, we have $\nabla . \vec{E} \neq 0$ and $\vec{J} \neq 0$ here. For an applied AC electric field of the form $\vec{E} = \vec{E}_0 e^{-i\omega t}$, the equation of motion of the electrons is plasma is given as

$$m_e \frac{d^2\vec{x}}{dt^2} = -\gamma \frac{d\vec{x}}{dt} - e\vec{E}.$$

Here γ is the damping coefficient of the medium which is a dominating factor for highly concentrated plasma, and therefore can be neglected for a dilute plasma ($\gamma \approx 0$). Therefore we have

$$m_e \frac{d^2\vec{x}}{dt^2} = -e\vec{E} = -e\vec{E}_0 e^{-i\omega t}$$

Assuming an oscillatory solution of the form $\vec{x} = \vec{x}_0 e^{-i\omega t}$ we get

$$-m_e \omega^2 \vec{x}_0 e^{-i\omega t} = -e\vec{E}_0 e^{-i\omega t}$$

$$\text{or } m_e \omega^2 \vec{x} = e\vec{E}_0 e^{-i\omega t}$$

$$\text{or } \vec{x} = \frac{e\vec{E}_0}{m_e \omega^2} e^{-i\omega t}$$

The drift velocity ($\vec{v}$) at any instant will be equal to

$$\vec{v} = \frac{d\vec{x}}{dt} = -\frac{ie\vec{E}_0}{m_e \omega} e^{-i\omega t} = -\frac{ie}{m_e \omega} \vec{E}$$

From Drude Formula, we get the current density ($\vec{J}$) as

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$$\vec{J} = -n_e e\vec{v} = \frac{in_e e^2}{m_e \omega}\vec{E}$$

Comparing this result with the known formula $\vec{J} = \sigma\vec{E}$, we get the expression for the conductivity (σ) of a dilute plasma as

$$\sigma(\omega) = \frac{in_e e^2}{m_e \omega} = \frac{i\varepsilon_0 {\omega_p}^2}{\omega}$$

where ω_p is the plasma frequency defined before. We find that the conductivity of a plasma is a *purely imaginary* quantity. We conclude that plasma is not a typical Ohmic conductor but a reactance. There is a phase shift by 90^0 between the electric field and the current density. The imaginary conductivity is due to the inertia of the electrons, where plasma behaves like an inductance.

EM Wave Propagation through Dilute Plasma:

One of the Maxwell Equations for this case becomes

$$\nabla \times \vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} = \mu_0\sigma\vec{E} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\text{or } \nabla \times \vec{B} = \frac{i\mu_0\varepsilon_0{\omega_p}^2}{\omega}\vec{E} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

Taking curl on both sides of the equation, and using the fact $\nabla.\vec{B} = 0$ we get

$$\nabla^2\vec{B} = \frac{i\mu_0\varepsilon_0{\omega_p}^2}{\omega}\frac{\partial\vec{B}}{\partial t} + \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}.$$

Similarly we get for the electric field

$$\nabla^2\vec{E} = \frac{i\mu_0\varepsilon_0{\omega_p}^2}{\omega}\frac{\partial\vec{E}}{\partial t} + \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}.$$

We look for a plane wave solution given by $\vec{E}(z,t) = \vec{E}_0 e^{i(kz-\omega t)}$, which gives us

$$-k^2 = \mu_0\varepsilon_0{\omega_p}^2 - \mu_0\varepsilon_0\omega^2$$

$$\text{or } k^2 = \frac{\omega^2 - {\omega_p}^2}{c^2}$$

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$$\text{or } \omega^2 = {\omega_p}^2 + c^2 k^2$$

This relation is known as the *Dispersion Relation* for a plasma. Here $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ is the speed of EM wave in free space or vacuum. One typical plasma dispersion plot has been shown in Fig. 2.

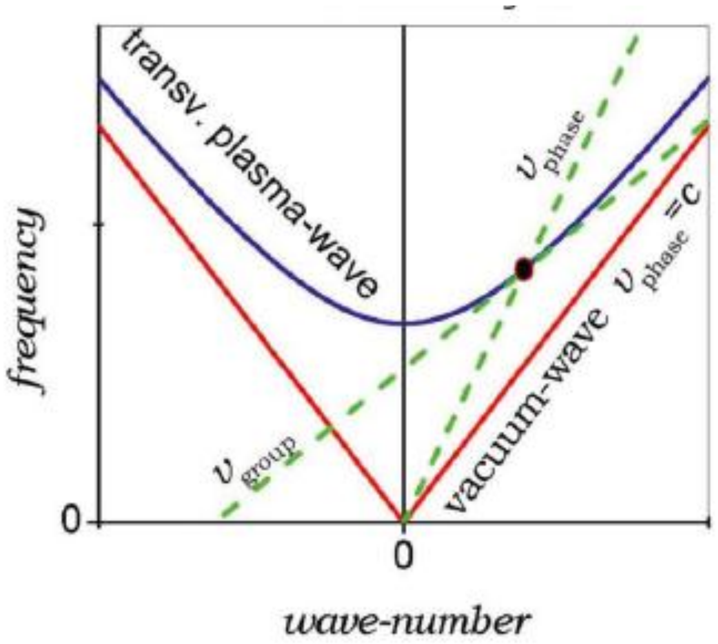

Fig. 2

The phase velocity (v_{phase}) is calculated as

$$v_{phase} = \frac{\omega}{k} = \frac{c\omega}{\sqrt{\omega^2 - {\omega_p}^2}}.$$

The group velocity (v_{group}) is calculated as

$$v_{group} = \frac{d\omega}{dk} = \frac{c\sqrt{\omega^2 - {\omega_p}^2}}{\omega}.$$

Here the refractive index (n) is calculated as $n = \frac{c}{v_{phase}} = \frac{\sqrt{\omega^2 - {\omega_p}^2}}{\omega}$.

Depending on the values of ω and ω_p we can have three possible cases,

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(a) For $\omega > \omega_p$, we find that the refractive index (n) is real. Since wave number k is also real, waves can propagate without damping. Plasma behaves as a *lossless dielectric* with a refractive index $n < 1$. Thus the phase velocity of waves exceeds the phase velocity of electromagnetic waves in vacuum, $v_{phase} > c$.

(b) For $\omega = \omega_p$, we have refractive index $n = 0$ and $k = 0$. That means, there is no wave, but only a plasma oscillation.

(c) For $\omega < \omega_p$, both n and k are imaginary. Let us consider $k = i\kappa$, where κ is expressed as $\frac{\sqrt{\omega_p{}^2-\omega^2}}{c}$. There is no wave propagation here. EM waves when penetrating a plasma will decay exponentially over a length scale known as the *attenuation length* (δ) given by

$$\delta = \frac{1}{\kappa} = \frac{c}{\sqrt{\omega_p{}^2-\omega^2}}.$$

It plays the same role as the skin depth for a metallic conductor. For small frequencies ($\omega \to 0$), the attenuation length $\delta \approx \frac{c}{\omega_p}$ becomes independent of the frequency.

EM Wave Propagation through Ionosphere:

The ionosphere exists between about 90 and 1000 km above the earth's surface. Radiation from the sun ionizes atoms and molecules here, liberating electrons from molecules and creating a space of free electron and ions. Subjected to an external electric field from a radio signal, these free and ions will experience a force and be pushed into motion. However, since the mass of the ions is much larger than the mass of the electrons, ionic motions are relatively small and will be ignored here. Free electron densities on the order of 10^{10} to 10^{12} electrons per cubic metre are produced by ionization from the sun's rays (we take typically $n_e \sim 8 \times 10^{11}$ m^{-3}). Layers of high densities of electrons are given special names called the D, E and F layers, as shown in Fig.

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3. During the day the F layer splits into two layers called the F_1 and F_2 layers, while the D layer vanishes completely at night.

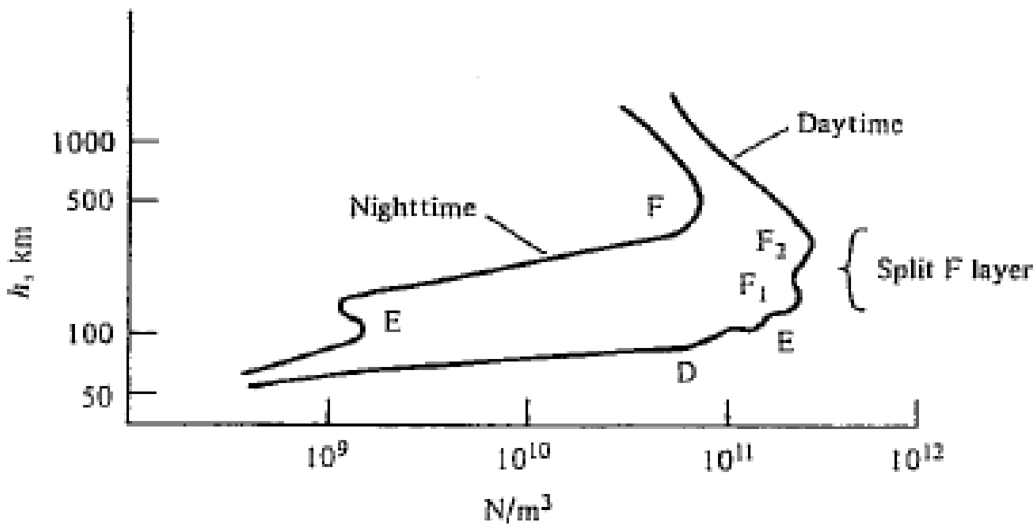

Fig. 3

Radio waves below 10 MHz are significantly affected by the ionosphere, primarily because radio waves in this frequency range are effectively reflected by the ionosphere. The E and F layers are the most important for this process. For frequencies beyond 10 MHz, the waves tend to penetrate through the atmosphere versus being reflected. The major usefulness of the ionosphere is that the reflections enable wave propagation over a much larger distance than would be possible with line-of-sight or even atmospheric refraction effects. This is shown graphically in Fig. 4.

Let us try to understand the facts from the theoretical background. For an electron concentration $n_e \sim 8 \times 10^{11}$ m^{-3}, the plasma frequency (ω_p) of the ionosphere becomes

$$\omega_p = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}} \approx 50.39 \times 10^6 \text{ s}^{-1}$$

$$\text{or } f_p = \frac{\omega_p}{2\pi} \approx 8 \text{ MHz.}$$

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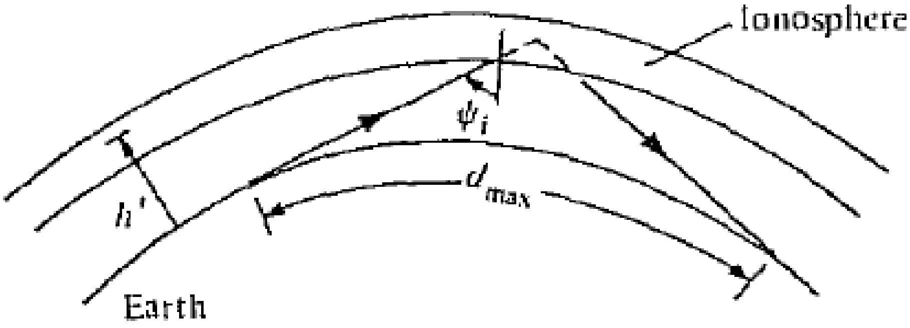

Fig. 4

AM radio waves having typical frequency $f < 1.6$ MHz get blocked, and therefore get reflected (via total internal reflection) by the ionosphere (since $f < f_p$). On the other hand, FM radio waves (having typical frequency $f \sim 20$ MHz) just pass through the ionosphere due to the fact $f > f_p$.

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(All the figures have been collected from the above mentioned references)

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