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DSC1BT (Electricity and Magnetism)

Topic – Electrostatics (Part – 1)

Introduction:

Two facts help us to explain the continuing importance of the classical description of electromagnetism in Modern Physics. Firstly, special relativity required no revision of classical electromagnetism. Historically speaking, special relativity grew out of classical electromagnetic theory and experiments inspired by it. Maxwell's equations, developed long before the work of Lorentz and Einstein, proved to be entirely compatible with relativity. Secondly, quantum modifications of the electromagnetic forces have turned out to be unimportant down to distances $< 10^{-12}$ meters, 100 times smaller than the atom. We can describe the repulsion and attraction of particles in the atom using the same laws that apply to the leaves of an electroscope, although we need quantum mechanics to predict how the particles will behave under those forces. For still smaller distances, a fusion of electromagnetic theory and quantum theory, called *quantum electrodynamics*, has been remarkably successful. Its predictions are confirmed by experiment down to the smallest distances yet to be explored.

Here in this e-report, we shall study the Physics of stationary electric charges, known as *electrostatics*. One fundamental property of electric charge is its existence in the two varieties that were long ago named *positive* and *negative*. The observed fact is that all charged particles can be divided into two classes such that all members of one class repel each other, while attracting members of the other class. If two small electrically charged bodies A and B, some distance apart, attract one another and if A attracts some third electrified body C, then we always find that B repels C. We clearly find a contrast here with gravitation, where there is only one kind of gravitational mass, and every mass attracts every other mass.

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Quantization of Charge:

The electric charges we find in nature come in units of one magnitude only, equal to the amount of charge carried by a single electron. We denote the magnitude of that charge by e with a value of 1.6×10^{-19} C (particularly when we are paying attention to sign, we write $-e$ for the charge on the electron itself). We have already noted that the positron carries precisely that amount of charge, as it must if charge is to be conserved when an electron and a positron annihilate, leaving nothing but light. What seems more remarkable is the apparently exact equality of the charges carried by all other charged particles, called the equality, for instance, of the positive charge on the proton and the negative charge on the electron. It has been established experimentally that the proton and the electron do not differ in magnitude of charge by more than 1 part in 10^{20}.

The equality is really exact for some reason not yet understood. It may be connected with the possibility, suggested by certain theories that a proton can, very rarely, decay into a positron and some uncharged particles. If that were to occur, even the slightest discrepancy between proton charge and positron charge would violate charge conservation. Several experiments designed to detect the decay of a proton have not yet registered with certainty a single decay. If and when such an event is observed, it will show that exact equality of the magnitude of the charge of the proton and the charge of the electron (the positron's antiparticle) can be regarded as a corollary of the more general law of charge conservation. That notwithstanding, we now know that the internal structure of all the strongly interacting particles called *hadrons*, a class that includes the proton and the neutron involves basic units called *quarks*, whose electric charges come in multiples of $e/3$. The proton, for example, is made with three quarks, two with charge $2e/3$ and one with charge $-e/3$. The neutron, on the other hand contains one quark with charge $2e/3$ and two quarks with charge $-e/3$. The fact of charge quantization lies outside the scope of classical electromagnetism, of course. Therefore, we will ignore it and act as if our point charges q could have any strength whatsoever.

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Coulomb's Law:

The experimenters of the eighteenth century knew that the objects could be charged with electricity so that they would attract or repel each other. Comparing these forces with the gravitational attraction, these pioneers must have argued about a similar law for the electrostatic forces. It was a French engineer, Charles A. Coulomb who announced an experimental verification of this law in 1785, which states that two stationary electric charges repel or attract one another with a force proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them.

We can state this compactly in mathematical form as

$$|\vec{F}_{21}| = F_{21} \propto \frac{q_1 q_2}{r^2}$$

$$\text{or vectorially } \vec{F}_{21} = k\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

Here q_1 and q_2 are numbers giving the magnitude and sign of the respective charges, r is the linear distance between these charges, $\hat{r}_{21}$ is the unit vector in the direction from charge q_1 to charge q_2 and $\vec{F}_{21}$ is the force acting on charge q_2 due to charge q_1 (shown in Fig. 1)

Fig. 1

Similarly, the force on charge q_1 due to charge q_2 is (using the fact $\hat{r}_{12} = -\hat{r}_{21}$)

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} = -k\frac{q_1 q_2}{r^2}\hat{r}_{21} = -\vec{F}_{21}$$

Of course we must assume, in writing the 1st equation that both charges are well localized, each occupying a region small compared with r. Otherwise we will not even define the distance r precisely.

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The value of the constant k in 1st equation depends on the units in which r, F, and q are to be expressed. In SI units, k is given by

$$k = 8.988 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

Instead of k, it is customary (for historical reasons) to introduce a constant ε_0 which is defined by

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.854 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$$

The constant ε_0 is known as the *electric permittivity of free space or vacuum*. In terms of ε_0, Coulomb's Law takes the form

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

Principle of Superposition. When there are more than two point charges, the total force on any one of them is the vector sum of the forces on it due to each charge considered separately. This is known as the principle of superposition. This is again an experimental fact.

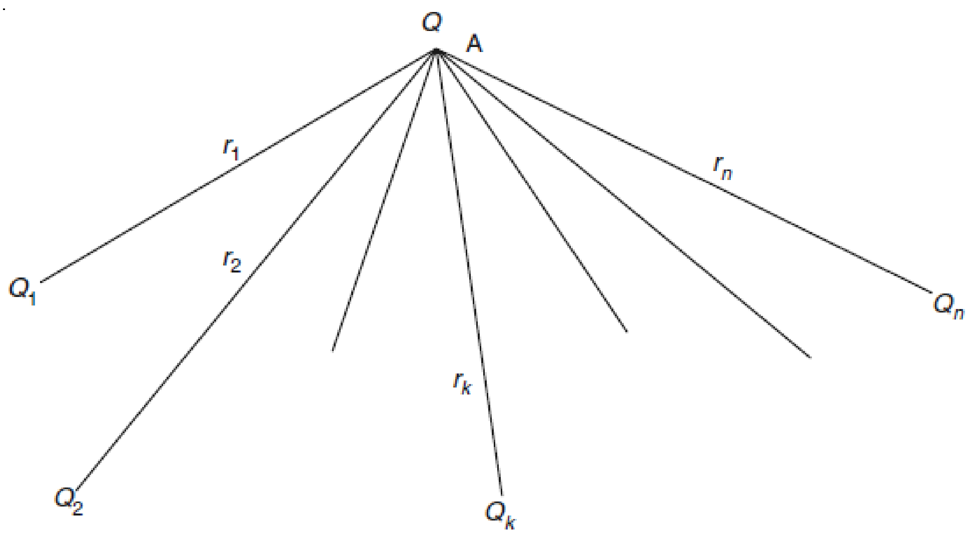

Fig. 2

Superposition means combining two sets of sources into one system by adding the second system on top of the first without altering the configuration of either one. Our principle ensures that the force on a charge placed at any point in the

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combined system will be the vector sum of the forces that each set of sources, acting alone, causes to act on a charge at that point. This principle must not be taken lightly for granted. There may well be a domain of phenomena, involving very small distances or very intense forces, where superposition no longer holds. Indeed, we know of quantum phenomena in the electromagnetic field that do represent a failure of superposition, seen from the viewpoint of the classical theory.

Thus if there are N point charges $q_1, q_2, . . . , q_N$ distributed in a region, then the force on the charge Q at a point A is given by (as shown in Fig. 2)

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Qq_1}{{r_1}^2}\hat{r}_1 + \frac{Qq_2}{{r_2}^2}\hat{r}_2 + \cdots + \frac{Qq_N}{{r_N}^2}\hat{r}_N\right) = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Qq_k}{{r_k}^2}\hat{r}_k$$

where r_k is the distance between the kth charge q_k and Q.

Electric Field:

Let us consider some arrangement of charges $q_1, q_2, . . . , q_N$, fixed in space, and we are interested not in the forces they exert on one another, but only in their effect on some other charge q_0 that might be brought into their vicinity. We know how to calculate the resultant force on this charge, given its position which we may specify by the coordinates x, y, z. The force on the charge q_0 is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_0 q_i}{{r_{0i}}^2}\hat{r}_{0i}$$

where $\vec{r}_{0i}$ is the vector from the ith charge in the system to the point (x, y, z). As we see, the force is proportional to q_0, so if we divide out q_0 we obtain a vector quantity that depends only on the structure of our original system of charges, $q_1, q_2, . . . , q_N$, and on the position of the point (x, y, z). We call this vector function as the *electric field* ($\vec{E}$) arising from the charges $q_1, q_2, . . . , q_N$ written as

$$\vec{E}(x, y, z) = \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{{r_{0i}}^2}\hat{r}_{0i}$$

It now can be generalized from *point charges* to *continuous charge distributions*. A volume distribution of charge is described by a scalar charge

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density function ρ, which is a function of position, with the dimensions of charge per unit volume. That means, ρ times a volume element gives the amount of charge contained in that volume element. If we write ρ as a function of the coordinates x, y, z, then $\rho(x, y, z)dxdydz$ is the charge contained in the little box of volume $dV = dxdydz$, located at the point (x, y, z).

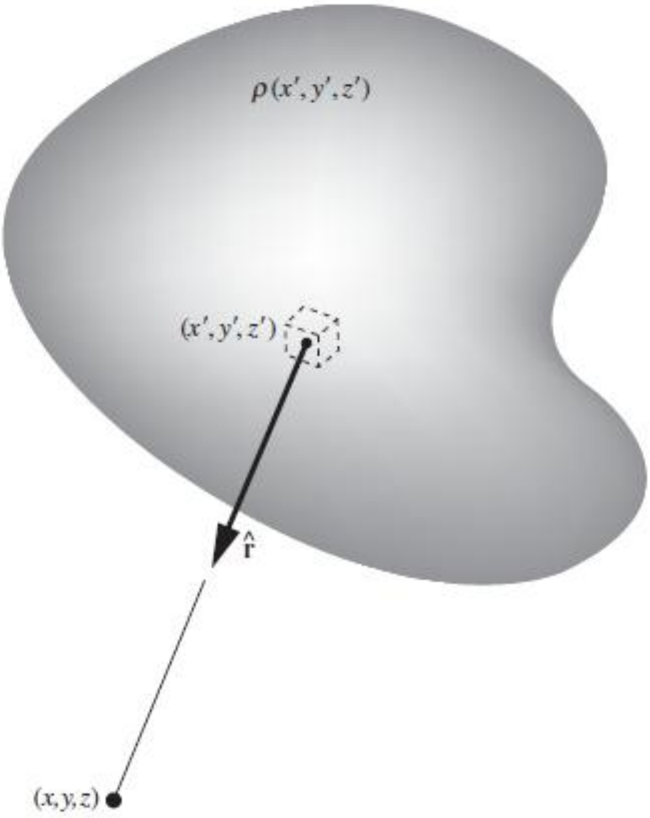

Fig. 3

On an atomic scale, of course, the charge density varies enormously from point to point. Still it proves to be a useful concept in that domain. However, we shall use it mainly when we are dealing with macroscale systems, so large that a volume element $dV = dxdydz$ can be quite small relative to the size of our system, although still large enough to contain many atoms or elementary charges. If the source of the electric field is to be a continuous charge distribution rather than point charges, we merely replace the sum in the case of discrete point charges with the appropriate integral. The integral gives the

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electric field at (x, y, z), which is produced by charges located at other points (x', y', z').

$$\vec{E}(x,y,z) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(x\prime,y\prime,z\prime)\hat{r}dx\prime dy\prime dz\prime}{r^2}$$

This is a volume integral. Holding (x, y, z) fixed, we let the variables of integration, x', y' and z', range over all space containing charge, thus summing up the contributions of all the bits of charge. The unit vector $\hat{r}$ points from the point (x', y', z') to (x, y, z), as shown in Fig. 3.

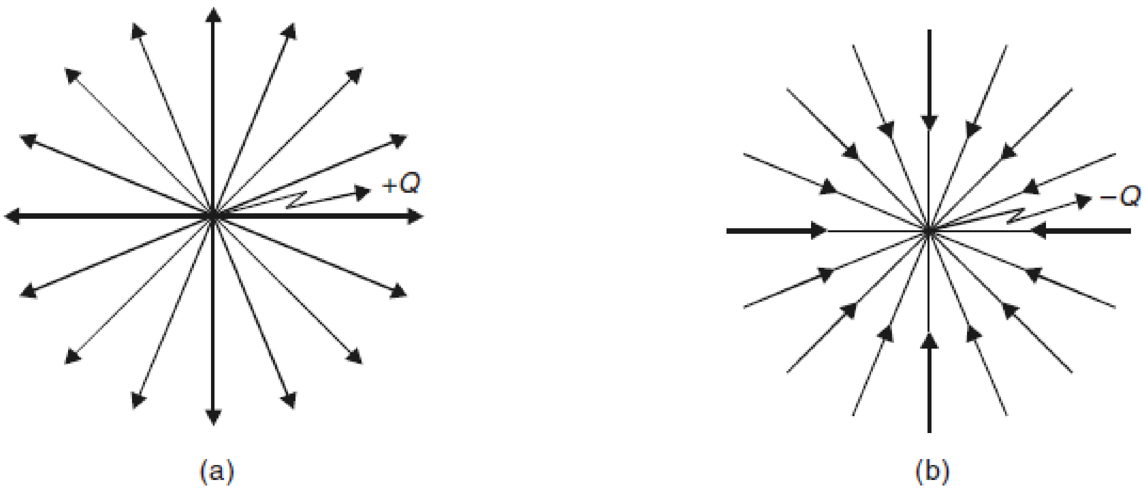

Fig. 4

Lines of Force. To visualize mentally the electric field, we use the concept of the lines of force. A line of force is a continuous curve, whose direction at every point is that of the vector $\vec{E}$. Hence the lines of force from a single point charge would be radial straight lines extending outwards. Lines of force for point charges are shown in Fig. 4. The whole space of the field is imagined to be filled with such lines and a charge brought into the field would experience a force whose direction is that of the line of force at that point where the extraneous charge is brought in. Here we meet with a conceptual difficulty. It should be noted that this intruding charge is not a passive device. It is by itself a source of electric force and the existing lines of force in its vicinity would get distorted by its presence. And yet when we say that the force on this extraneous charge is $Q\vec{E}$, the value of $\vec{E}$ to be used is that value which the electric force

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would have if the intruding charge were not there. The point we should note is that as a method of calculating the force on the charged particle, this works correctly, but as a picture of what is happening in the electric field it is unsatisfactory.

Electric Flux:

The relation between the electric field and its sources can be expressed in a remarkably simple way, one that we shall find very useful. For this we need to define a quantity called *electric flux.*

Let us consider some electric field in space and in this space some arbitrary closed surface, like a balloon of any shape. Fig. 5 shows such a surface, the field being suggested by a few field lines. Now let us divide the whole surface into little patches that are so small that over any one patch the surface is practically flat and the vector field does not change appreciably from one part of a patch to another. The area of a patch has a certain magnitude in square meters, and a patch defines a unique direction, which is the outward-pointing normal to its surface.

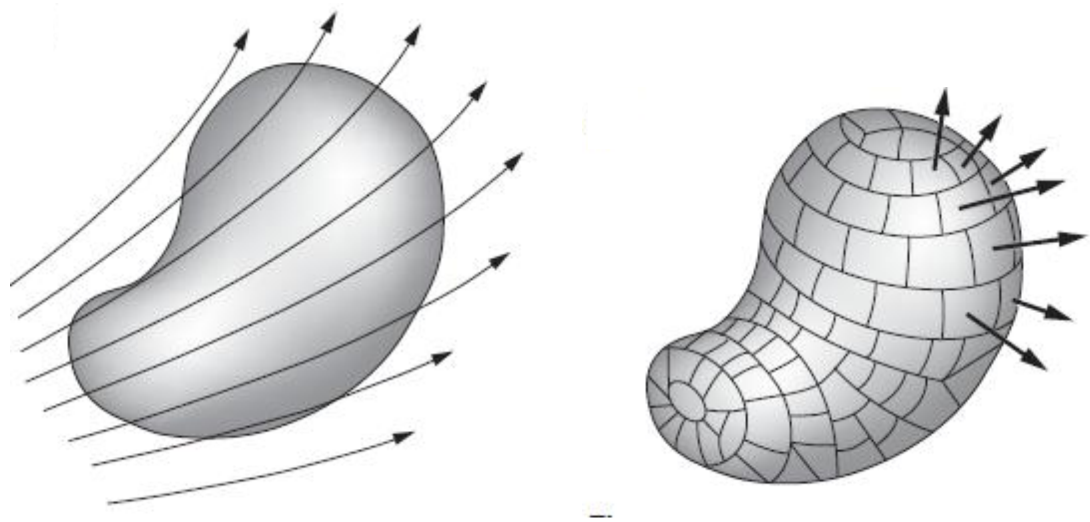

Fig. 5

Let this magnitude and direction be represented by a vector. Then for every patch into which the surface has been divided, such as patch number j, we have a vector $\vec{a}_j$ giving its area and orientation. Note that the vector $\vec{a}_j$ does not depend at all on the shape of the patch, it doesn't matter how we have divided up the surface, as long as the patches are small enough. Let $\vec{E}_j$ be the electric field vector at the location of patch number j. The scalar product $\vec{E}_j.\vec{a}_j$ is known as the *flux* through that bit of surface.

To understand the origin of the name, let us imagine a vector function that represents the velocity of motion in a fluid, where the velocity varies from one place to another but is constant in time at any one position. Denote this vector field by $\vec{v}$, measured in meters/second. Then, if $\vec{a}$ is the oriented area in square meters of a frame lowered into the water, $\vec{v}.\vec{a} = va\cos\theta$ is the *rate of flow* of water through the frame per second (Fig. 6). The $\cos\theta$ factor in the standard expression for the dot product correctly picks out the component of $\vec{v}$ along the direction of $\vec{a}$, or equivalently the component of $\vec{a}$ along the direction of $\vec{v}$. We must emphasize that our definition of flux is applicable to any vector function, whatever physical variable it may represent.

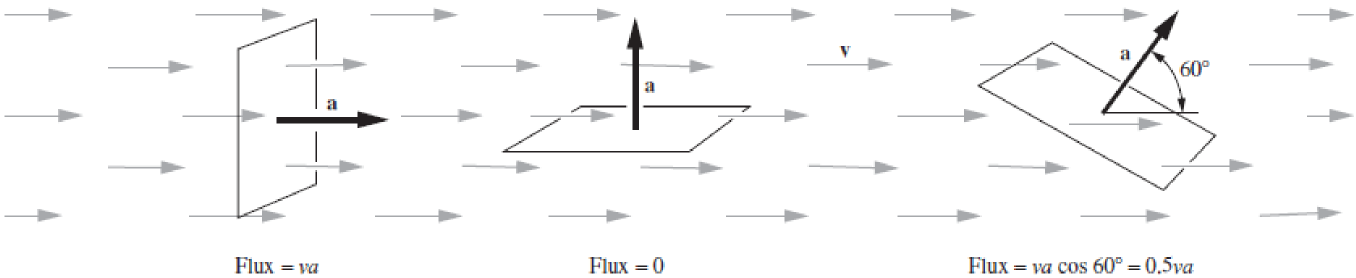

Fig. 6

Now from Fig. 5, let us add up the flux through all the patches to get the flux through the entire surface, a scalar quantity which we shall denote by Φ

$$\Phi = \sum_j \vec{E}_j.\vec{a}_j$$

Letting the patches become smaller and more numerous without limit, we pass from the sum in the previous equation to a surface integral, given as

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$$\Phi = \int \vec{E}.d\vec{S}$$

A surface integral of any vector function $\vec{F}$, over any arbitrary surface S, means just the following concept. One has to divide S into small patches, each represented by a vector outward, of magnitude equal to the patch area and at every patch, the scalar product of the patch area vector and the local $\vec{F}$ needs to be taken. Then all these products need to be summed up, and the limit of this sum, as the patches shrink, is the surface integral.

This concludes part 1 of this e-report.

The discussion will be continuing in the part 2 of this e-report.

Reference(s):

Electricity and Magnetism, E.M. Purcell and D.J. Morin, Cambridge University Press

Electromagnetism: Theory and Applications, Ashutosh Pramanik, PHI

(All the figures have been collected from the above mentioned references)