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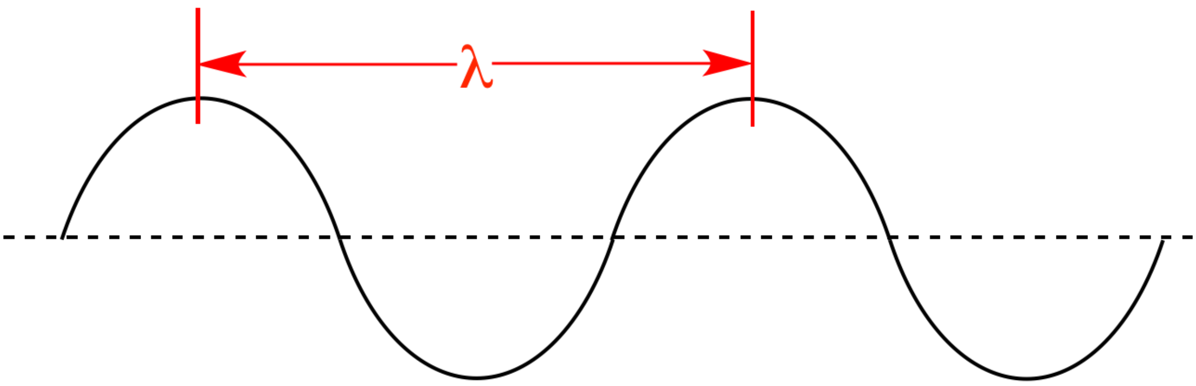
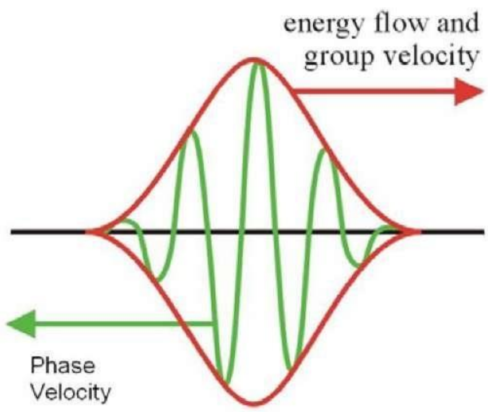
Topic:

Wave Motion & Velocity of Waves: Pressure of a Longitudinal Wave. Energy Transport. Intensity of Wave. Water Waves: Ripple and Gravity Waves, Particle and Wave Velocities.

References:

(i) *Waves & Oscillations-B. Ghosh*
(ii) *Waves- A. B. Gupta*
(iii) *Waves and Oscillations, -N. Subrahmanyam & Brij Lal (Author)*





Acoustic Pressure

Let the equation of a plane wave propagating in an elastic medium along +x direction is,

$$\psi = a \sin \frac{2\pi}{\lambda}(ct - x)$$

Let p be the excess pressure.

$$\therefore p = -E\frac{d\psi}{dx} = \frac{2\pi}{\lambda} aE \cos \frac{2\pi}{\lambda}(ct-x) \quad \text{——(1)}$$

$$p_{max} = \frac{2\pi a E}{\lambda}$$

$$\therefore p = p_{max} \cos \frac{2\pi}{\lambda}(ct - x)$$

$\therefore$ Mean squared value of pressure $= \frac{1}{T}\int_0^T \left(\frac{2\pi a E}{\lambda}\right)^2 \cos^2 \frac{2\pi}{\lambda}(ct-x)\,dt$

$$= \frac{1}{T}\left(\frac{2\pi a E}{\lambda}\right)^2 \cdot \frac{T}{2}$$

$$= 2 \left(\frac{\pi a E}{\lambda}\right)^2$$

$\therefore$ Root mean square pressure $p_{rms} = \sqrt{2}\cdot\frac{\pi a E}{\lambda}$

$$= \sqrt{2}\cdot\frac{\pi a}{\lambda}\rho c^2$$

$$\left[\because c = \sqrt{E/\rho}\right]$$

$$\therefore p_{rms} = \sqrt{2}\cdot \pi a \rho n^2 \lambda \qquad [\text{Since } c = \lambda n]$$

$$= \sqrt{2}\cdot \pi a \rho n c$$

Energy density of Plane progressive wave

(a) Kinetic Energy:

The K.E of an element of thickness δx & unit cross sectional area is

$$E_K = \frac{1}{2}\rho\, \delta x \left(\frac{\partial \psi}{\partial t}\right)^2 \qquad \rho = \text{density of medium}$$

We have,

$$\psi = a \sin\frac{2\pi}{\lambda}(ct - x) \qquad \text{——— (1)}$$

$$\therefore \frac{\partial \psi}{\partial t} = \frac{2\pi a c}{\lambda} \cos\frac{2\pi}{\lambda}(ct - x)$$

$$\therefore E_K = \frac{1}{2}\rho\,\delta x \cdot \frac{4\pi^2 a^2 c^2}{\lambda^2} \cos^2\frac{2\pi}{\lambda}(ct - x)$$

Avg. K.E $= \overline{E_K}$

$$= \frac{1}{T}\int_0^T \frac{1}{2}\rho\,\delta x \cdot \frac{4\pi^2 a^2 c^2}{\lambda^2} \cdot \cos^2\frac{2\pi}{\lambda}(ct - x)\, dt$$

$$= \frac{1}{T}\frac{2\pi^2 \rho a^2 c^2}{\lambda^2}\delta x \cdot \frac{T}{2}$$

$$= \frac{\pi^2 \rho a^2 c^2}{\lambda^2} \cdot \delta x$$

(b) Potential Energy.

The force acting on the element of thickness δx and of unit cross sectional area is given by,

$$F = \text{mass} \times \text{acceleration}$$

$$= \rho\,\delta x \times \frac{\partial^2 \psi}{\partial t^2}$$

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Differentiating (1) w.r.t. t we get,

$$\frac{\partial^2 \psi}{\partial t^2} = -\frac{4\pi^2 c^2}{\lambda^2} a \sin \frac{2\pi}{\lambda}(ct - x)$$

$$\therefore \quad F = -\rho.\delta x.\frac{4\pi^2 c^2}{\lambda^2}.a \sin \frac{2\pi}{\lambda}(ct - x)$$

$$= -\frac{4\pi^2 c^2 \rho}{\lambda^2}.\delta x\, \psi$$

The negative sign indicates that F is directed towards the mean position opposite to the displacement. Hence work done to produce a small displacement is given by.

$$dE_p = \frac{4\pi^2 c^2 \rho}{\lambda^2}.\delta x.\psi\, d\psi$$

$\therefore$ Total work done $\quad E_p = \dfrac{4\pi^2 c^2 \rho}{\lambda^2}\,\delta x \displaystyle\int_0^{\psi} \psi\, d\psi$

$$= \frac{2\pi^2 c^2 \rho}{\lambda^2}.\delta x.\psi^2$$

$$= \frac{2\pi^2 c^2 \rho}{\lambda^2}.\delta x.a^2 \sin^2 \frac{2\pi}{\lambda}(ct - x)$$

This work is stored in the volume of the medium as potential energy of the wave.

Avg. potn Energy ~~$E_p = \frac{\rho \lambda^2 c^2}{\lambda^2}$~~

$$E_p = \frac{2\pi^2 c^2}{\lambda^2} \rho\, \delta x\, \frac{a^2}{T} \int_0^T \sin^2 \frac{2\pi}{\lambda}(ct - x)\, dt$$

$$= \frac{2\pi^2 c^2}{\lambda^2} \rho\, \delta x\, \frac{a^2}{T} \cdot \frac{T}{2}$$

$$= \frac{\pi^2 c^2}{\lambda^2} \cdot \rho a^2 \cdot \delta x$$

$\therefore$ The avg. K.E = avg. P.E

$\therefore$ The total Energy of the element of thickness δx and unit cross section is given by,

$$E = \frac{\pi^2 c^2}{\lambda^2} \rho a^2 \delta x + \frac{\pi^2 c^2}{\lambda^2} \rho a^2 \delta x$$

$$= \frac{2\pi^2 c^2}{\lambda^2} \rho a^2 \delta x$$

(c) Energy density

Energy density = energy per unit volume

$$= \frac{2\pi^2 c^2 \rho a^2}{\lambda^2} \delta x \Big/ \delta x$$

$$= \frac{2\pi^2 c^2 \rho a^2}{\lambda^2}$$

We have, $c = n\lambda$

$\therefore$ $\boxed{\text{Energy density} = 2\pi^2 n^2 \rho a^2}$

So, the energy density is proportional to the density of the medium and to the squares of amplitude and frequency of the wave.

The Potn. energy at a point is maximum when $\sin\frac{2\pi}{\lambda}(ct-x)=1$ or, $\psi=a$

$$\therefore \boxed{E_P^{max} = 2\pi^2 n^2 \rho a^2}$$

The K.E at a point is maximum when $\cos\frac{2\pi}{\lambda}(ct-x)=1$ or, $\sin\frac{2\pi}{\lambda}(ct-x)=0$ ie, $\psi=0$

This means that when particle is passing through its mean position.

$\therefore$ The value of the maximum Kinetic Energy $\boxed{E_K^{max} = 2\pi^2 n^2 a^2 \rho}$

Intensity of the wave.

The intensity of the wave in a specific direction at a point is defined as average rate of flow of energy across unit area normal to that direction at the point.

If, δA is the area & δw is the avg. rate of flow of energy we have avg. intensity $I_A = \frac{\delta w}{\delta A}$

$$\therefore I = \lim_{\delta A \to 0} \frac{\delta w}{\delta A} = \frac{dw}{dA}$$

$I =$ energy density $\times$ velocity of the wave

$$= 2\pi^2 n^2 a^2 \rho c$$

We have, $P_{rms} = \sqrt{2}\, \pi a \rho n c$

$$\therefore I = \frac{P_{rms}^2}{\rho c}$$

Bel & Decibel:

The subjective sensation of loudness, is however not proportional to intensity but depends upon intensity. The range of audible frequency is from about 20 cycles/sec to 20,000 cy/sec and the sensitivity of normal ear is different at different frequencies. The subjective sensation of loudness at a frequency is given by Weber-Fechner Law. This law states that the change in loudness (ΔL) is directly proportional to the ratio between the change in intensity (ΔI) to the original intensity I.

$$\therefore \quad \Delta L = K\frac{\Delta I}{I} \qquad K = \text{Const}^n.$$

Integrating we get,

$$L = k \ln I + C_1$$

$$L = 2.303\, k \log_{10} I + C_1 \qquad [C_1 = \text{Const}^n]$$

Hence, $L_2 - L_1 = 2.303 k \log_{10}\frac{I_2}{I_1} = c \log_{10} I_2/I_1$ —(i)

L_1 & L_2 are loudness level corresponding to the intensities I_1 & I_2, respectively.

The threshold audibility, that is, the lower limit of audibility, for a note of 1000 Hz corresponding to intensity 10^{-12} watt/m^2 & pressure $= 2\times10^{-5}$ N/m^2.

This is taken as standard intensity I_0

If, $I_1 = I_0$ & $\frac{I_2}{I_0} = 10$ then the

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Source of sound having intensity I_2 is said to be 1 bel or 10 decibels.

$\therefore$ Intensity levels in bels $= \log_{10} \frac{I}{I_0}$

Intensity levels in decibels $= 10 \log_{10} I/I_0$

Phon & Sone:

When we measure the loudness, the sound under investigation and the standard reference tone are alternately listened to by the observer from the same distance. The intensity of the reference tone is then gradually increased till the two sounds appear to be equally loud. The increase in intensity in decibels of the reference tone from its initial value is the measures in 'phon' of the equivalent loudness of the sound under investigation.

Now a days loudness is measured in sone. The loudness produced by a pure tone of frequency 1000 Hz above 40 decibels of the lower limit of audibility is called sone.

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Frequently Asked Questions:

1. Derive the equation of pressure of plane progressive wave.
2. Derive the equation of energy density of plane progressive wave.
3. Derive the relation between intensity and energy density of plane progressive wave.
4. Write short note on Bel, Decidel, Phon, Sone.

Link to Audio visual Lectures (e-Lectures) and NPTEL lectures on this topic given by Distinguish Professors of Indian & Foreign Universities:

(1) *https://nptel.ac.in/courses/115/106/115106119/*
(2) *https://nptel.ac.in/courses/122/105/122105023/*
(3) *https://nptel.ac.in/courses/115/105/115105083/*
(4) *https://onlinecourses.nptel.ac.in/noc19_ph18/preview*
(5) *https://nptel.ac.in/courses/115/106/115106090/*
(6) *https://www.youtube.com/watch?v=hXiMBRDtyFM*
(7) *https://www.digimat.in/nptel/courses/video/115106119/L01.html*