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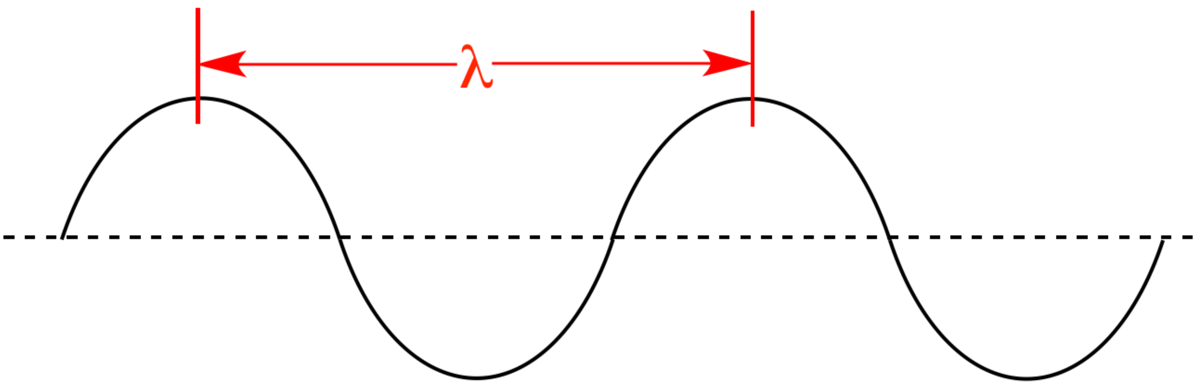
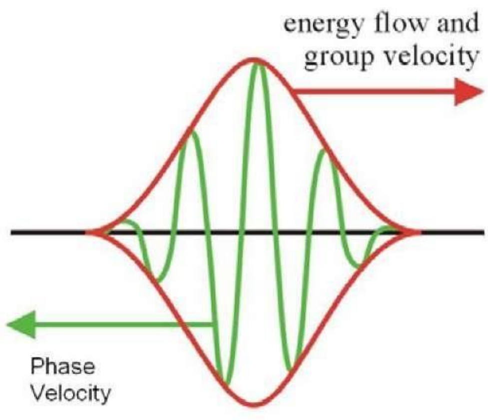
Topic:

Wave Motion & Velocity of Waves: Plane Progressive (Travelling) Waves, Wave Equation, Differential Equation, Velocity of Longitudinal Waves in a Fluid in a Pipe,. Newton's Formula for Velocity of Sound, Laplace's Correction, Velocity of Longitudinal Waves in a solid

References:

(i) Waves & Oscillations-B. Ghosh
(ii) Waves- A. B. Gupta
(iii) Waves and Oscillations, -N. Subrahmanyam & Brij Lal (Author)





Progressive wave.

A wave propagating from one point to another in a medium without being subjected to any boundary condition, is termed as progressive wave.

Characteristics

(i) The waves are produced by the periodic vibrations of the particles in the medium.

(ii) Each particle of the medium oscillates about its mean position with the same amplitude and period.

(iii) The vibrations are simple harmonic

(iv) The phase difference between two vibrating particles on the line of propagation is proportional to the distance between the particles.

(v) The wave velocity in a given medium is a constant determined by the density and the elasticity of the medium.

Equation of plane progressive wave

Let a constant progressive wave propagate in the positive x-axis. ψ be the displacement of a particle at any instant t.

$\therefore$ Simplest type of S.H. vibration $\psi(t) = a\sin\omega t$

a = amplitude & ω = angular frequency

If λ be the wave length. Hence vibration of a particle at a distance x from an arbitrary origin on line of wave propagation in the forward direction lags in phase angle

$= \frac{2\pi}{\lambda}x$

$\therefore$ Displacement $\psi(x,t) = a\sin\left(\omega t - \frac{2\pi}{\lambda}x\right)$

We have, $\omega = \frac{2\pi}{T} = 2\pi n = \frac{2\pi c}{\lambda}$

$\therefore$ $\psi(x,t) = a\sin\left(\frac{2\pi c}{\lambda}t - \frac{2\pi}{\lambda}x\right)$

$$\boxed{\psi(x,t) = a\sin\frac{2\pi}{\lambda}(ct - x)}$$

If we have. $\psi(t) = a\cos\omega t$

$$\boxed{\psi(x,t) = a\cos\frac{2\pi}{\lambda}(ct - x)}$$

If wave travelling in the (-ve) direction.

$$\boxed{\begin{aligned}\psi(x,t) &= a\sin\frac{2\pi}{\lambda}(ct + x)\\ \psi(x,t) &= a\cos\frac{2\pi}{\lambda}(ct + x)\end{aligned}}$$

Differential wave equation in plane progressive wave in one dimension:

We have, $\psi(x,t) = a e^{i\frac{2\pi}{\lambda}(ct-x)}$

$= a e^{ik(ct-x)}$ ——— (1)

where, $k = \frac{2\pi}{\lambda}$ = propagation constn.

Differentiating,

$\therefore \frac{\partial \psi}{\partial t} = \frac{2\pi i c}{\lambda} a e^{i\frac{2\pi}{\lambda}(ct-x)} = \frac{2\pi i c}{\lambda}\psi(x,t)$

Again Differentiating

$$\frac{\partial^2 \psi}{\partial t^2} = -\frac{4\pi^2 c^2}{\lambda^2}\psi(x,t) \quad \text{——— (2)}$$

Again Differentiating equn. (1) w.r.t. x we get,

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{4\pi^2}{\lambda^2}\psi(x t)$$

$$\therefore c^2 \frac{\partial^2 \psi}{\partial x^2} = -\frac{4\pi^2 c^2}{\lambda^2}\psi(x,t) \quad \text{——— (3)}$$

Compairing equn. (2) & (3) we get,

$$\boxed{\frac{\partial^2 \psi}{\partial t^2} = c^2 \frac{\partial^2 \psi}{\partial x^2}}$$

Alternative treatment

Let $\psi(x,t) = a \sin(\omega t - kx)$ be the equation of plane progressive wave.

Diff. twice w.r.t. t we get.

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 a \sin(\omega t - kx) \quad \text{— (2)}$$

Diff twice w.r.t x we get.

$$\frac{\partial^2 \psi}{\partial x^2} = -k^2 a \sin(\omega t - kx) \quad \text{— (3)}$$

Combining (2) & (3) we get.

$$\boxed{\frac{\partial^2 \psi}{\partial t^2} = c^2 \frac{\partial^2 \psi}{\partial x^2}}$$

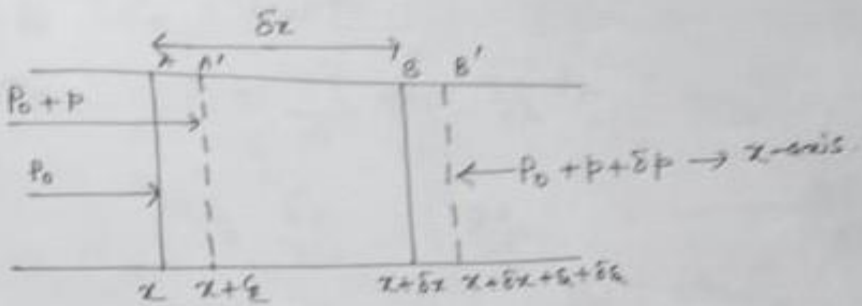
The three dimensional wave eqn. is

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 . \nabla^2 \psi$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial t^2} = c^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right)$$

** For solution of 1D, 3D, plane progressive wave see seperation of variable note in Mathematical Physics part.

Velocity of plane longitudinal sound waves in a fluid:



Let us consider a tube of fluid of unit cross-section with its axis in the direction of propagation of the wave and two plane sections A and B of the tube at x and $x+\delta x$. Here, $\delta x >> \lambda$

The volume of slice $AB = \delta x$.

Under the influence of the sound wave particles of A displaced to A' with a distance ξ & those B displaced by $\xi + \delta\xi$ to B'.

$\therefore$ Final volume of $A'B' =$

$$(x+\delta x+\xi+\delta\xi) - (x+\xi) = \delta x + \delta\xi = \delta x + \frac{\partial \xi}{\partial x}\delta x$$

$$\therefore \Delta = \frac{\text{Change in vol}^{m}}{\text{original vol}^{m}} = \frac{\left(\delta x + \frac{\partial \xi}{\partial x}.\delta x\right) - \delta x}{\delta x} = \frac{\partial \xi}{\partial x}$$

Let, P_0 be the pressure on A & p be the excess pressure on A'.

$\therefore$ Excess pressure on $B' = p + \delta p = p + \frac{\partial p}{\partial x}\delta x$

$\therefore$ Resultant force on slice $= p - \left(p + \frac{\partial p}{\partial x}\delta x\right)$

$$= -\frac{\partial p}{\partial x}\delta x$$

If ρ_0 = density

Then, $\rho_0 \delta x \frac{\partial^2 \xi}{\partial t^2} = -\frac{\partial p}{\partial x}\delta x$

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = -\frac{\partial p}{\partial x} \qquad \text{———(2)}$$

Now, bulk modulus $(B) = \frac{\text{Vol}^m \text{ stress}}{\text{Vol}^m \text{ strain}}$

$$= \frac{P}{-\Delta}$$

$$\therefore p = -B\frac{\partial \xi}{\partial x} \qquad \text{———(3)}$$

Substituting the value of (3) in eqn (2)

we get, $\frac{\partial^2 \xi}{\partial t^2} = B/\rho_0 \frac{\partial^2 \xi}{\partial x^2}$

$$\boxed{\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial^2 \xi}{\partial x^2}}$$

where, $c = \sqrt{B/\rho_0}$

$\therefore$ $c = \sqrt{B/\rho_0}$, the velocity of propagation of sound waves.

Velocity of Sound wave in gases.

We have, $PV^{\gamma} = \text{Const}^{n}$.

Differentiating partially we get,

$$V^{\gamma}.dP + \gamma P V^{\gamma-1} dV = 0$$

$$\gamma P = -\frac{dP}{dV/V} = B$$

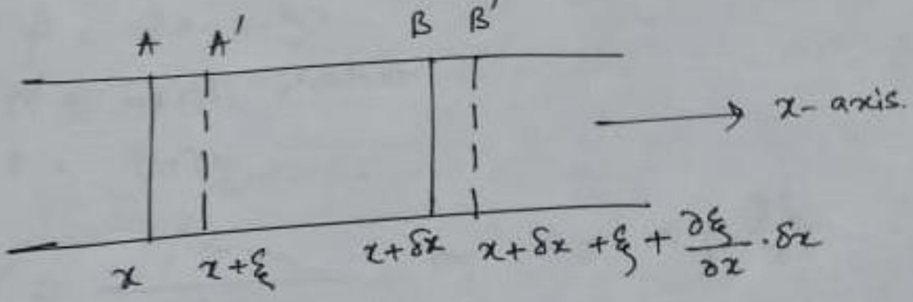
Therefore, The velocity of sound in gas

is $= c = \sqrt{B/\rho_0} = \sqrt{\gamma P/\rho_0}$ ——— (1)

We have, $PV = RT$ for perfect gas.

$\therefore \; c = \sqrt{\dfrac{\gamma RT}{M}}$ [Putting the value of P in eqn (1)]

Velocity of Longitudinal waves in a Solid bar



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Let us consider a solid bar, whose length is large compared to its lateral dimension.

Let us consider an elementary slice AB in the rod bounded by two planes A & B, $\perp^r$ to the length of the bar at x & $x+\delta x$. Suppose under the influence of the longitudinal waves propagating along the bar the plane A is displaced to A' by ξ.

$\therefore$ Displacement at B $= \xi + \frac{\partial \xi}{\partial x} \cdot \delta x$

New distance A'B' between the plane is $= \delta x + \frac{\partial \xi}{\partial x} \delta x$

$$\therefore \text{Longitudinal strain} = \frac{\left(\delta x + \frac{\partial \xi}{\partial x} \cdot \delta x\right) - \delta x}{\delta x}$$

$$= \frac{\partial \xi}{\partial x}$$

If Y = Young modulus,
ρ = density
α = Cross section.
F = Force

$$\therefore Y = \frac{F/\alpha}{\partial \xi/\partial x} \quad \text{or,} \quad F = \alpha Y \frac{\partial \xi}{\partial x} \qquad \text{—— (1)}$$

The streaching force on the face at B

$$= F + \frac{\partial F}{\partial x} \delta x$$

$$= \alpha Y \frac{\partial \xi}{\partial x} + \alpha Y \frac{\partial^2 \xi}{\partial x^2} \cdot \delta x$$

$\therefore$ Resultant force,

$$\left(\alpha Y \frac{\partial \xi}{\partial x} + \alpha Y \frac{\partial^2 \xi}{\partial x^2} \delta x \right) - \alpha Y \frac{\partial \xi}{\partial x}$$

$$= \alpha Y \frac{\partial^2 \xi}{\partial x^2} \delta x \quad \text{——— (2)}$$

$$\therefore (\rho \alpha \delta x) \cdot \frac{\partial^2 \xi}{\partial t^2} = \alpha Y \frac{\partial^2 \xi}{\partial x^2} \delta x$$

$$\Rightarrow \boxed{\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial^2 \xi}{\partial x^2}} \quad \text{——— (3)}$$

where, $c = \sqrt{Y/\rho}$

Hence, $c = \sqrt{Y/\rho}$ is the velocity of longitudinal wave travelling along the rod.

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Frequently Asked Questions:

1. Derive the equation of plane progressive wave & its solution.
2. Derive the velocity of longitudinal wave in solid.
3. Derive the velocity of longitudinal wave in liquid.
4. Derive the velocity of plane progressive wave through gas.

Link to Audio visual Lectures (e-Lectures) and NPTEL lectures on this topic given by Distinguish Professors of Indian & Foreign Universities:

(1) https://nptel.ac.in/courses/115/106/115106119/
(2) https://nptel.ac.in/courses/122/105/122105023/
(3) https://nptel.ac.in/courses/115/105/115105083/
(4) https://onlinecourses.nptel.ac.in/noc19_ph18/preview
(5) https://nptel.ac.in/courses/115/106/115106090/
(6) https://www.youtube.com/watch?v=hXiMBRDtyFM
(7) https://www.digimat.in/nptel/courses/video/115106119/L01.html