COMPILED & CIRCULATED BY
Dr. Tapanendu Kamilya
Assistant Professor, Department of Physics, Narajole Raj College

Topic:

Maxwell Equations: Maxwell's equations. Displacement Current, Boundary Conditions at Interface between Different Media. Wave Equations, Plane Waves in Dielectric Media, Pointing Theorem and Pointing Vector.

Maxwell's Equations

With macroscopic view, we analyze the problems of electromagnetic analysis by solving the *Maxwell's equations* subject to certain boundary conditions.

Maxwell's equations are a set of equations, formulated or written in differential or integral form, stating the relationships among the basic along with fundamental electromagnetic quantities. These quantities are: (i) Electric field intensity (**E**), (ii) Electric displacement or electric flux density (**D**), (iii) **Magnetic** field intensity (**H**), (iv) Magnetic flux density (**B**), (v) Current density (**J**), (vi) Electric charge density (**ρ**)

For general time-varying fields, Maxwell's equations differential form can be written as:

$\nabla \cdot \mathbf{D} = \rho$	(1)	Gauss' Law
$\nabla \cdot \mathbf{B} = 0$	(2)	Gauss' Law for magnetism
$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	(3)	Faraday's Law
$\nabla \times \mathbf{H} = \dfrac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$	(4)	Ampère-Maxwell Law

The first and second equations are two forms of *Gauss' law*: the *Gauss' law* in electrostatics electric and *Gauss' law* in magnetostatics, respectively. The third and fourth equations are also referred to as *Faraday's law* and *Maxwell's Modified–Ampère's law*, respectively.

COMPILED & CIRCULATED BY
Dr. Tapanendu Kamilya
Assistant Professor, Department of Physics, Narajole Raj College

Name	Equation	
	Integral form	Differential form
Faraday's law of induction	$\oint_c \vec{E}\cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Ampère-Maxwell law	$\oint_c \vec{H}\cdot d\vec{l} = \iint_S \vec{J}\cdot d\vec{S} + \iint_S \frac{\partial \vec{D}}{\partial t}\cdot d\vec{S}$	$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$
Gauss' electric law	$\oiint_S \vec{D}\cdot d\vec{S} = \iiint_V \rho\, dV$	$\nabla \cdot \vec{D} = \rho$
Gauss' magnetic law	$\oiint_S \vec{B}\cdot d\vec{S} = 0$	$\nabla \cdot \vec{B} = 0$

Formulation in SI units

Name	Integral equations	Differential equations	Meaning
Gauss's law	$\oiint_{\partial\Omega} \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_{\Omega} \rho\, dV$	$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$	The electric field leaving a volume is proportional to the charge inside.
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B}\cdot d\mathbf{S} = 0$	$\nabla\cdot\mathbf{B} = 0$	There are no magnetic monopoles; the total magnetic flux piercing a closed surface is zero.
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot d\boldsymbol{\ell} = -\frac{d}{dt}\iint_{\Sigma} \mathbf{B}\cdot d\mathbf{S}$	$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	The voltage accumulated around a closed circuit is proportional to the time rate of change of the magnetic flux it encloses.
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0\iint_{\Sigma} \mathbf{J}\cdot d\mathbf{S} + \mu_0\varepsilon_0\frac{d}{dt}\iint_{\Sigma} \mathbf{E}\cdot d\mathbf{S}$	$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0\frac{\partial \mathbf{E}}{\partial t}\right)$	Electric currents and changes in electric fields are proportional to the magnetic field circulating about the area they pierce.

Equation of Continuity

We know that $\iint \mathrm{J.ds} = -\frac{dq}{dt}$

$$q = \iiint \rho\, dv$$

$$\iint \mathrm{J.ds} = -\frac{d\iiint \rho\, dv}{dt}$$

$\iint \mathrm{J.ds} = -\iiint \frac{\partial \rho}{\partial t} dv$

$\iint \mathrm{J.ds} = \iiint \boldsymbol{\nabla}.\boldsymbol{J}\, dv$

$\iiint \boldsymbol{\nabla}.\boldsymbol{J}\, dv = -\iiint \frac{\partial \rho}{\partial t} dv$

$$\iiint (\nabla.\boldsymbol{J} + \frac{\partial \rho}{\partial t}) dv = 0$$

$$\nabla.\boldsymbol{J} + \frac{\partial \rho}{\partial t} = 0$$

This is the required equation for the equation of continuity.

For, stationary current, $\frac{\partial \rho}{\partial t} = 0$at all points

Therefore, $\nabla.\boldsymbol{J} = 0$

Derivation of Maxwell's Equation

1. **Proof of $\nabla.\boldsymbol{D} = \rho$**

Let us consider a surface S bounding a volume V in a dielectric medium. If, ρ and ρ_p are the charge densities of free charge and polarization charges

$$\iint E.dS = \frac{1}{\varepsilon_0}\int_V (\rho + \rho_P) dv$$

$$\rho_P = -\nabla.\boldsymbol{P}$$

$$\iint E.dS = \frac{1}{\varepsilon_0}\int_V (\rho - \nabla.\boldsymbol{P}) dv$$

$$\iint \varepsilon_0 E.dS = \int_V \rho\, dV - \int_V \nabla.\boldsymbol{P}\, dv$$

$$\int_V \nabla.(\varepsilon_0 \boldsymbol{E} + \boldsymbol{P})\, dv = \int_V \rho\, dV$$

$\varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \boldsymbol{D}$**,** The electric displacement vector = D

Therefore, $\int_V \nabla.\boldsymbol{D}\, dv = \int_V \rho\, dV$

Therefore, $\nabla.\boldsymbol{D} = \rho$

2. **Proof of $\nabla.\boldsymbol{B} = 0$**

Since the isolated magnetic poles and magnetic currents due to them have no physical significance

$\iint \boldsymbol{B}.\boldsymbol{dS} = 0$ where, B = magnetic induction vector

Therefore, $\nabla.\boldsymbol{B} = 0$

3. **Proof of $\nabla X E = -\frac{\partial B}{\partial t}$**

According to Faraday's law of electromagnetic induction, we know e.m.f. (e)

$$e = -\frac{d\varphi}{dt}$$

We know $\iint \boldsymbol{B}.\boldsymbol{dS} = \varphi$

Therefore,
$e = -\frac{d\iint \boldsymbol{B}.\boldsymbol{dS}}{dt} = \iint_s \ -\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}.\mathrm{d}\mathbf{S}$

We know, $\int \boldsymbol{E}.\boldsymbol{dl} = e$

Therefore, $\int \boldsymbol{E}.\boldsymbol{dl} = \iint_s \ -\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}.\,\mathrm{d}\mathbf{S}$

$$\int \nabla \boldsymbol{X}\boldsymbol{E}.\boldsymbol{dS} = \iint_s \ -\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}.\,\mathrm{d}\mathbf{S}$$

$$\iint_s \ \left(\nabla \boldsymbol{X}\boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}\right)\boldsymbol{dS} = \mathbf{0}$$

Therefore, $\left(\nabla \boldsymbol{X}\boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}\right) = \mathbf{0}$

Hence, $\nabla X E = -\frac{\partial B}{\partial t}$

4. **Proof of $\nabla X H = J + \frac{\partial D}{\partial t}$**

Maxwell's postulates: Displacement current

Form, Ampere's circuital law, we know that, $\int \mathbf{H}.\boldsymbol{dl} = \boldsymbol{I}$

And, $\int \mathbf{J}.\boldsymbol{dS} = \boldsymbol{I}$

$$\int \mathbf{H}.\boldsymbol{dl} = \int \mathbf{J}.\boldsymbol{dS}$$

$$\int \nabla \boldsymbol{X}\boldsymbol{H}.\boldsymbol{dS} = \int \mathbf{J}.\boldsymbol{dS}$$

$$\iint_s \ (\nabla \boldsymbol{X}\boldsymbol{H} - \boldsymbol{J}).\boldsymbol{dS} = \mathbf{0}$$

Therefore, $(\nabla \boldsymbol{X}\boldsymbol{H} = \boldsymbol{J})$

$$((\nabla).\nabla \boldsymbol{X}\boldsymbol{H} = \mathbf{0})$$

Therefore, $\nabla.\boldsymbol{J} = 0$

However, from equation of continuity, $\nabla.\boldsymbol{J} + \frac{\partial \rho}{\partial t} = 0$

Equation of continuity is a true phenomenon. Therefore, $\boldsymbol{\nabla}.\boldsymbol{J} = 0 \; is \; impossible$

Maxwell's assumed that the total current density $C = J + J_d$, $J_d = Displacement \; current$

Therefore, $\nabla.(\nabla X H) = \nabla.(J + J_d)$

$$\nabla.(J + J_d) = 0$$

$$\nabla.J = -\nabla.(J_d)$$

$$\nabla.J_d = \frac{\partial \rho}{\partial t}$$

But, $\boldsymbol{\nabla}.\boldsymbol{D} = \rho$

Hence, $\nabla.J_d = \frac{\partial(\boldsymbol{\nabla}.\boldsymbol{D})}{\partial t} = \nabla.(\frac{\partial D}{\partial t})$

Therefore, $J_d = \frac{\partial D}{\partial t}$

$$\nabla X H = J + J_d$$

$$\boldsymbol{\nabla X H} = \boldsymbol{J} + \frac{\boldsymbol{\partial D}}{\boldsymbol{\partial t}}$$

This is called Maxwell's modification on Ampere's law.

Maxwell's Equation in Integral form

1. $\boldsymbol{\nabla}.\boldsymbol{D} = \rho$

$$\int_v \nabla.D \, dV = \int_V \rho \, dV$$

$$\int_S D.dS = \int_V \rho \, dV$$

2. $\boldsymbol{\nabla}.\boldsymbol{B} = 0$

$$\int_v \nabla.B \, dV = 0$$

$$\int_S B.dS = 0$$

3. $\boldsymbol{\nabla X E} = -\frac{\boldsymbol{\partial B}}{\boldsymbol{\partial t}}$

$\int_s (\nabla X E).dS$ = - = $-\int_s \frac{\partial B}{\partial t}.dS$

$$Therefore, \int \mathbf{E}.\mathbf{dl} = -\frac{\partial}{\partial t}\iint_S \mathbf{B}.\mathbf{dS}$$

$$This\ is\ Maxwell's\ third\ equation\ in\ integral\ form.$$

4. $\nabla X \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$

$$\int_s (\nabla XH).dS = \int_s (\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}).dS$$

$$Therefore, \int \mathbf{H}.\mathbf{dl} = \int_s (\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}).dS$$

$$\int \mathbf{H}.\mathbf{dl} = m.m.f\ (Magnetomotive\ Force)$$

Maxwell's Equation in some particular cases

Maxwell's equation in free space

In free space, the volume charge density $\rho = 0$ and current density $J = 0$ Hence Maxwell's equation takes the form of-

$$\nabla.\boldsymbol{D} = 0$$

$$\nabla.\boldsymbol{B} = 0$$

$$\nabla \mathrm{X} \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}$$

$$\nabla \mathrm{X} \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial \boldsymbol{t}}$$

Here, $\boldsymbol{D} = \varepsilon_0 \boldsymbol{E}$ and $\boldsymbol{B} = \mu_0 \boldsymbol{H}$

Maxwell's Equation in Linear isotropic medium

Here, $\boldsymbol{D} = \varepsilon \boldsymbol{E}$ **and** $\boldsymbol{B} = \mu \boldsymbol{H}$

Therefore, Maxwell's equations become-

$$\nabla.\boldsymbol{E} = {}^{\rho}/_{\varepsilon}$$

$$\nabla.\boldsymbol{B} = 0$$

$$\nabla \mathrm{X} \boldsymbol{E} = \boldsymbol{\mu}\frac{\partial \boldsymbol{H}}{\partial t}$$

$$\nabla \mathrm{X} \boldsymbol{H} = \boldsymbol{J} + \varepsilon\frac{\partial \boldsymbol{E}}{\partial t}$$

COMPILED & CIRCULATED BY
Dr. Tapanendu Kamilya
Assistant Professor, Department of Physics, Narajole Raj College

Electromagnetic Energy: Poynting Theorem:

The electromagnetic potential energy $U_c = \frac{1}{2}\int_V \boldsymbol{E}.\boldsymbol{D}\, dv$

The energy stored in magnetic field $U_m = \frac{1}{2}\int_V \boldsymbol{H}.\boldsymbol{B}\, dv$

$$\nabla.\boldsymbol{D} = \rho$$

$$\nabla.\boldsymbol{B} = 0$$

$$\nabla \text{X} \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \text{X} \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

Therefore, $\boldsymbol{H}.(\nabla\, X\, \boldsymbol{E}) = -\boldsymbol{H}.\frac{\partial \boldsymbol{B}}{\partial t}$

$$\boldsymbol{E}.(\nabla\, X\, \boldsymbol{H}) = \boldsymbol{E}.\boldsymbol{J} + \boldsymbol{E}.\frac{\partial \boldsymbol{D}}{\partial t}$$

$\boldsymbol{H}.(\nabla\, X\, \boldsymbol{E}) - \boldsymbol{E}.(\nabla\, X\, \boldsymbol{H}) = -(\boldsymbol{H}.\frac{\partial \boldsymbol{B}}{\partial t} + \boldsymbol{E}.\frac{\partial \boldsymbol{D}}{\partial t}) - \boldsymbol{E}.\boldsymbol{J}$

Or, $\nabla.(\boldsymbol{E}\, X\, \boldsymbol{H}) = -(\boldsymbol{H}.\frac{\partial \boldsymbol{B}}{\partial t} + \boldsymbol{E}.\frac{\partial \boldsymbol{D}}{\partial t}) - \boldsymbol{E}.\boldsymbol{J}$

Here, $\boldsymbol{D} = \varepsilon \boldsymbol{E}$ **and** $\boldsymbol{B} = \mu \boldsymbol{H}$

Here, $\boldsymbol{E}.\frac{\partial \boldsymbol{D}}{\partial t} = E.\frac{\partial}{\partial t}(\varepsilon E) = \frac{1}{2}\varepsilon \frac{\partial}{\partial t} E^2 = \frac{\partial}{\partial t}(\frac{1}{2}\boldsymbol{E}.\boldsymbol{D})$

Here, $\boldsymbol{H}.\frac{\partial \boldsymbol{B}}{\partial t} = H.\frac{\partial}{\partial t}(\mu H) = \frac{1}{2}\mu \frac{\partial}{\partial t} H^2 = \frac{\partial}{\partial t}(\frac{1}{2}\boldsymbol{H}.\boldsymbol{B})$

$$\nabla.(\boldsymbol{E}\, X\, \boldsymbol{H}) = -\frac{\partial}{\partial t}[\frac{1}{2}(\boldsymbol{E}.\boldsymbol{D} + \boldsymbol{H}.\boldsymbol{B}) - \boldsymbol{J}.\boldsymbol{E}$$

$$\int_V \nabla.(\boldsymbol{E}\, X\, \boldsymbol{H}) dV = -\int_V \{\frac{\partial}{\partial t}[\frac{1}{2}(\boldsymbol{E}.\boldsymbol{D} + \boldsymbol{H}.\boldsymbol{B})\} dV - \int_V \boldsymbol{J}.\boldsymbol{E}\, dV$$

$$\int_S (\boldsymbol{E}\, X\, \boldsymbol{H}).d\boldsymbol{S} = -\frac{\partial}{\partial t}\int_V \frac{1}{2}(\boldsymbol{E}.\boldsymbol{D} + \boldsymbol{H}.\boldsymbol{B})\, dV - \int_V \boldsymbol{J}.\boldsymbol{E}\, dV$$

Therefore, $-\int_V \boldsymbol{J}.\boldsymbol{E}\, dV = \frac{\partial}{\partial t}\int_V \frac{1}{2}(\boldsymbol{E}.\boldsymbol{D} + \boldsymbol{H}.\boldsymbol{B})\, dV + \int_S (\boldsymbol{E}\, X\, \boldsymbol{H}).d\boldsymbol{S}$

$$-\int_V \boldsymbol{J}.\boldsymbol{E}\, dV \;\; represents\ rate\ of\ energy\ transferred\ into\ the\ electromagnetic\ field\ through\ the\ motion\ of\ free\ charge\ in\ volume\ V.$$

The physical meaning of the equation is that-

the time rate of change of electromagnetic energy with a certain volume plus time rate of energy flowing out through the boundary surface is equal to the power transferred into the electromagnetic field.

The term $\frac{\partial}{\partial t}\int_V \frac{1}{2}(\boldsymbol{E}.\boldsymbol{D}+\boldsymbol{H}.\boldsymbol{B})\,dV$ represents the rate of electromagnetic energy stored in volume V.

The term, $\int_S (\boldsymbol{E}\ X\ \boldsymbol{H}).d\boldsymbol{S}$ ***represents energy flow per unit time per unit area***

S=$\boldsymbol{E}\ X\ \boldsymbol{H}$ is known **as Poynting vector and is interpreted as power flux i.e. amount of energy crossing unit area placed perpendicular to the vector, per unit time.**

Wave Equation:

Let us consider a uniform linear medium having permittivity ε, permeability μ, and conductivity σ but not any charge or any current other than determined by Ohm's law.

Here, $\boldsymbol{D} = \varepsilon\boldsymbol{E}$ **and** $\boldsymbol{B} = \mu\boldsymbol{H}$ **and** $\boldsymbol{J} = \sigma\boldsymbol{E}$ **and** $\boldsymbol{\rho} = \boldsymbol{0}$

So, Maxwell's equation becomes-

$$\nabla.\boldsymbol{D} = \rho$$

$$\nabla.\boldsymbol{B} = 0$$

$$\nabla \mathrm{X}\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \mathrm{X}\boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

$$\nabla.\boldsymbol{E} = 0$$

$$\nabla.\boldsymbol{H} = 0$$

$$\nabla \mathrm{X}\boldsymbol{E} = -\mu\frac{\partial \boldsymbol{H}}{\partial t}$$

$$\nabla \mathrm{X}\boldsymbol{H} = \sigma\boldsymbol{E} + \varepsilon\frac{\partial \boldsymbol{E}}{\partial t}$$

$$\nabla\ X(\nabla X\boldsymbol{E}) = -\boldsymbol{\mu}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}(\nabla X\boldsymbol{H})$$

$$\nabla\ X\ (\nabla X\boldsymbol{E}) = -\boldsymbol{\mu}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}(\sigma\boldsymbol{E} + \varepsilon\frac{\partial \boldsymbol{E}}{\partial t})$$

$$\nabla\ X\ (\nabla X\boldsymbol{E}) = -\boldsymbol{\mu}\sigma\frac{\partial \boldsymbol{E}}{\partial t} - \boldsymbol{\varepsilon\mu}\frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

$\nabla X (\nabla X \boldsymbol{H}) = -\mu\sigma \frac{\partial H}{\partial t} - \varepsilon\mu \frac{\partial^2 H}{\partial t^2}$

We know that $\nabla X (\nabla X \boldsymbol{H}) = \nabla(\nabla . \boldsymbol{H}) - \nabla^2 \boldsymbol{H}$

$$\nabla X (\nabla X \boldsymbol{E}) = \nabla(\nabla . \boldsymbol{E}) - \nabla^2 \boldsymbol{E}$$

$$\nabla . \boldsymbol{H} = 0$$

$$\nabla . \boldsymbol{E} = 0$$

The equation becomes-

$\nabla^2 \boldsymbol{E} - \mu\sigma \frac{\partial E}{\partial t} - \varepsilon\mu \frac{\partial^2 E}{\partial t^2} = 0$

$\nabla^2 \boldsymbol{H} - \mu\sigma \frac{\partial H}{\partial t} - \varepsilon\mu \frac{\partial^2 H}{\partial t^2} = 0$

Plane electromagnetic wave in free space:

Maxwell's equation are-

$$\nabla . \boldsymbol{D} = \rho$$

$$\nabla . \boldsymbol{B} = 0$$

$$\nabla X \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla X \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

In free space σ =0 and $\boldsymbol{\rho} = \mathbf{0}$; $\boldsymbol{\mu} = \boldsymbol{\mu_0}$ and $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon_0}$

Maxwell's equation becomes-

$$\nabla . \boldsymbol{E} = 0$$

$$\nabla . \boldsymbol{H} = 0$$

$$\nabla X \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t}$$

$$\nabla X \boldsymbol{H} = \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

$\nabla X (\nabla X E) = -\mu_0 \frac{\partial}{\partial t}(\varepsilon_0 \frac{\partial E}{\partial t})$

$$\nabla X (\nabla X E) = -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

We know that $\nabla X (\nabla X \boldsymbol{H}) = \nabla(\nabla . \boldsymbol{H}) - \nabla^2 \boldsymbol{H}$

$$\nabla X (\nabla X \boldsymbol{E}) = \nabla(\nabla.\boldsymbol{E}) - \nabla^2 \boldsymbol{E}$$

$$\nabla.\boldsymbol{H} = 0$$

$$\nabla.\boldsymbol{E} = 0$$

The equation becomes-

$\nabla^2 \boldsymbol{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \mathbf{0}$………(1)

$\nabla^2 \boldsymbol{H} - \mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{H}}{\partial t^2} = \mathbf{0}$…….(2)

In general wave equation-

$\nabla^2 \boldsymbol{u} - \mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{u}}{\partial t^2} = \mathbf{0}$

$\nabla^2 \boldsymbol{u} = \frac{1}{c^2} \frac{\partial^2 \boldsymbol{u}}{\partial t^2}$…………(3)

$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \text{ X } 10^8 \frac{m}{sec}$= **velocity of light**

Plane electromagnetic waves in a Non-conducting Isotropic medium:

Let us consider a uniform linear medium having permittivity ε, permeability μ, and conductivity σ but not any charge or any current other than determined by Ohm's law.

Here, $\boldsymbol{D} = \varepsilon \boldsymbol{E}$ **and** $\boldsymbol{B} = \mu \boldsymbol{H}$ **and** $\boldsymbol{J} = \sigma \boldsymbol{E} = \mathbf{0}$ **and** $\boldsymbol{\rho} = \mathbf{0}$

So, Maxwell's equation becomes-

$$\nabla.\boldsymbol{D} = \rho$$

$$\nabla.\boldsymbol{B} = 0$$

$$\nabla \text{X} \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \text{X} \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

$$\nabla.\boldsymbol{E} = 0$$

$$\nabla.\boldsymbol{H} = 0$$

$$\nabla \text{X} \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t}$$

$$\nabla \text{X} \boldsymbol{H} = \varepsilon \frac{\partial \boldsymbol{E}}{\partial t}$$

$$\nabla X(\nabla X E) = -\mu \frac{\partial}{\partial t}(\nabla X H)$$

$$\nabla X (\nabla X E) = -\mu \frac{\partial}{\partial t}(\varepsilon \frac{\partial E}{\partial t})$$

$$\nabla X (\nabla X E) = -\varepsilon\mu \frac{\partial^2 E}{\partial t^2}$$

$$\nabla X (\nabla X H) = -\varepsilon\mu \frac{\partial^2 H}{\partial t^2}$$

We know that $\nabla X (\nabla X H) = \nabla(\nabla . H) - \nabla^2 H$

$$\nabla X (\nabla X E) = \nabla(\nabla . E) - \nabla^2 E$$

$$\nabla . H = 0$$

$$\nabla . E = 0$$

The equation becomes-

$$\nabla^2 E - \varepsilon\mu \frac{\partial^2 E}{\partial t^2} = 0$$

$$\nabla^2 H - \varepsilon\mu \frac{\partial^2 H}{\partial t^2} = 0$$

$$\nabla^2 u - \varepsilon\mu \frac{\partial^2 u}{\partial t^2} = 0$$

$$\nabla^2 u = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \ldots\ldots\ldots\ldots(3)$$

$$c = \frac{1}{\sqrt{\varepsilon\mu}}$$

Frequently Asked Questions:

1. Starting from Maxwell's equation, establish Coulomb's law?
2. Show that the equation of continuity is contained in Maxwell's equation.
3. Starting from 3rd and 4th Maxwell's equation show that $\nabla . B = 0 \; and \; \nabla . E = \rho$
4. Write down the Maxwell's Equations?
5. Derive Maxwell's four equations?
6. Derive the Maxwell's equation in integral form.
7. State and prove the equation of continuity.
8. Write down Maxwell's equation in free space and linear isotropic medium.
9. What is displacement current?
10. Derive Maxwell's modification on Ampere's Law. Hence, derive Maxwell's 3rd equation.

11. What is pointing vector? State and proof the Pointing theorem for the flow of energy in electromagnetic wave.
12. Derive the electromagnetic wave equation in free space. Hence, find the velocity of electromagnetic wave in free space.
13. Derive the general wave equations of electromagnetic wave.
14. Derive the equation of plane electromagnetic waves in a Non-conducting Isotropic medium and find the velocity of wave.

References:

(i) *ELECTROMAGNETIC THEORY AND ELECTRODYNAMICS*, ***Author: Satyaprakash, Published by Kedarnath Ramnath (2009 Ed.)***

(ii) *Introduction to Electrodynamics*, ***Author- D. J. Griffiths, Published by Cambridge India (4th Ed.).***

(iii) ***https://physics.stackexchange.com/ (Image is taken)***

(iv) ***https://physics.stackexchange.com/questions/256739/what-are-the-differences-between-the-differential-and-integral-forms-of-e-g-ma (Image is taken)***

Link to Audio visual Lectures (e-Lectures) and NPTEL lectures on this topic given by Distinguish Professors of Indian & Foreign Universities:

(1) *https://nptel.ac.in/courses/115/101/115101005/*

(2) *https://nptel.ac.in/courses/115/101/115101004/*

(3) *https://nptel.ac.in/courses/115/104/115104088/*

(4) *https://nptel.ac.in/courses/108/104/108104087/*

(5) *https://nptel.ac.in/courses/115/106/115106122/*

(6) *https://www.digimat.in/nptel/courses/video/108104087/L01.html*

(7) *https://onlinecourses.nptel.ac.in/noc19_ph08/preview*