



## COMPILED & CIRCULATED BY Dr. TapanenduKamilya

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### *Topic:*

**Boolean Algebra:** De Morgan's Theorems. Boolean Laws. Simplification of Logic Circuit using Boolean Algebra. Fundamental Products. Idea of Minterms and Maxterms. Conversion of a Truth table into Equivalent Logic Circuit by (1) Sum of Products Method and (2) Karnaugh Map. NAND and NOR Gates as Universal Gates

## Boolean Algebra

### Laws of Boolean algebra:

$$A + 0 = A$$

$$A + 1 = 1$$

$$A + A = A$$

$$\bar{A} + A = 1$$

$$A \cdot 0 = 0$$

$$A \cdot 1 = A$$

$$A \cdot A = A$$

$$\bar{\bar{A}} \cdot A = 0$$

$$\bar{\bar{A}} = A$$

### Commutative laws:

$$A + B = B + A$$

$$A \cdot B = B \cdot A$$

### Associative laws:

$$A + (B + C) = (A + B) + C$$

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

### Distributive laws:

$$A \cdot (B + C) = A \cdot B + A \cdot C$$



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### De Morgan's Theorem:

(a) It states that the complement of a sum of variables is equal to the product of the complements of the individual variables.

$$\overline{A + B + C + \dots} = \overline{A} \cdot \overline{B} \cdot \overline{C} \dots$$

(b) The second theorem states that the complement of a product of variables is equal to the sum of the complements of the individual variables.

$$\overline{A \cdot B \cdot C \dots} = \overline{A} + \overline{B} + \overline{C} + \dots$$

### **Simplification of Boolean Expression**

$$AB + \overline{A}C + BC$$

$$AB + \overline{A}C + BC$$

$$= AB + \overline{A}C + BC \cdot 1$$

$$= AB + \overline{A}C + BC(A + \overline{A})$$

$$= AB + \overline{A}C + BCA + BC\overline{A}$$

$$= (AB + ABC) + (\overline{A}C + BC\overline{A})$$

$$= AB(1 + C) + \overline{A}C(1 + B)$$

$$= AB + \overline{A}C$$

1.  $ABC + \overline{A}CB + A\overline{B}C + \overline{A}B\overline{C} + \overline{A}C\overline{B} + \overline{A}B\overline{C}$

2.  $\overline{\overline{A}B} + AB + \overline{A}$

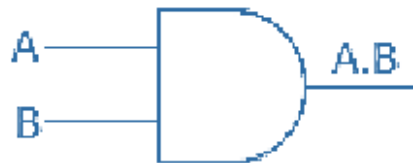
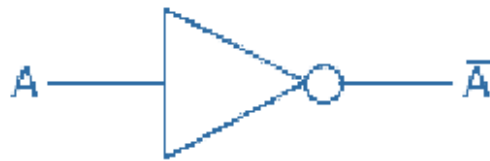
3.  $ABC + \overline{A}CB + A\overline{B}C + AB\overline{C}$

4.  $AB + \overline{A}B + \overline{A}B$



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**Symbol of AND, OR, NOT Gates**



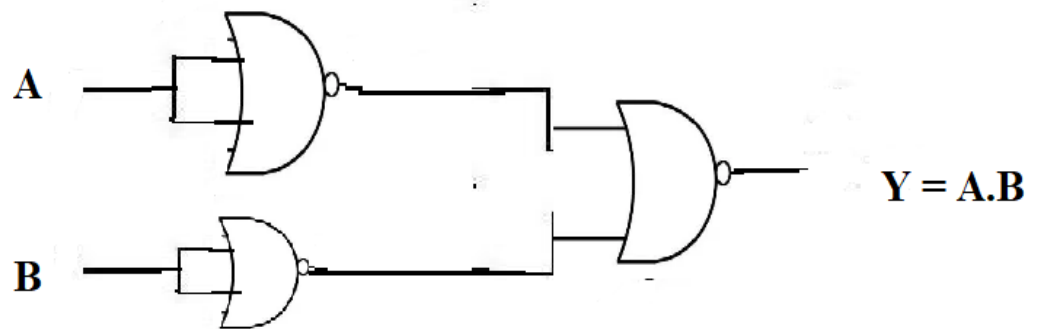
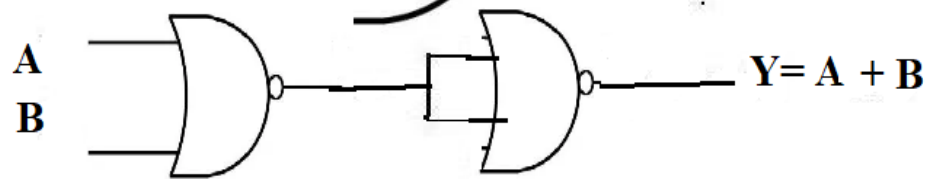
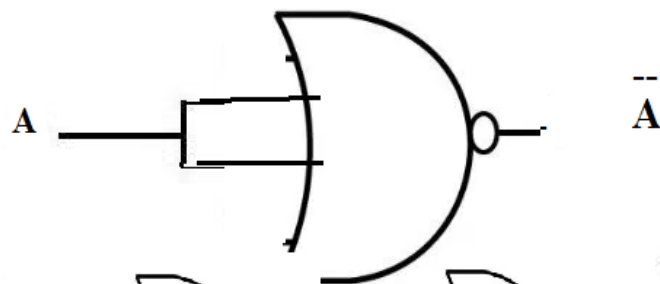
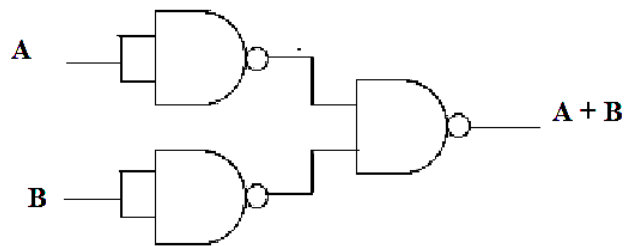
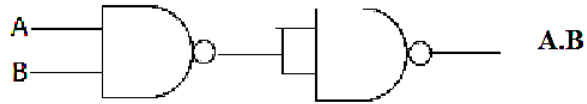
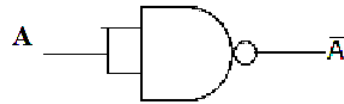
**NAND and NOR Gates are called Universal Gates**

All basic gates like AND, OR, NOT can be prepared by NAND and NOR Gates. Therefore, these two gates are called Universal gates.



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### KARNAUGH MAP

The Karnaugh map is a pictorial method of grouping together terms with common factors and therefore removing unwanted variables and simplification of Boolean Expression. The Karnaugh maps are typically known as special arrangements of a truth tables.

The SOP (Sum of Product) and POS (Product of Sum) are termed as for the methods for realizing the particular logic function. The SOP encompasses the OR of the multiple product terms. In opposition, POS yields of the AND of the multiple sum terms.

Minterm of a Boolean expression refers 1 for the output of a single cell, and 0s for all other cells in a Karnaugh map, or truth table. On the other hand, the Maxterm of Boolean expression refers 0 for the output of a single cell expression, and 1s for all other cells in the Karnaugh map, or truth table.

#### Steps to solve expression using K-map-

1. At first one have to select the **K-map** according to the number of variables.
2. Then the identification of minterms or maxterms will be done as given in problem.
3. Putting of 1's in blocks of **K-map** will be done respective to the minterms (0's elsewhere) for SOP.
4. Put 0's in blocks of **K-map** will be done respective to the maxterms (1's elsewhere) for POS.

#### Minimization of Boolean Functions using K-Maps

Follow these **rules for simplifying K-maps** in order to get standard sum of products form.

- ❖ We have to select the respective K-map based on the number of variables present in the Boolean function.
- ❖ We have to place the ones at respective min term cells in the K-map for the Boolean expression which is given in the form of sum of min terms.
- ❖ We have to place the ones (1) in all possible cells of K-map for which the Boolean expression is given in the form of product terms.



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- ❖ Then we have to check for all the possibilities of grouping maximum number of adjacent ones. It should be the powers of two. We have to start grouping from the highest power of two. Then we have to find up to the least power of two. The number of variables considered in K-map is equal to the highest power whereas the least power is zero.
- ❖ Each grouping will provide either an accurate or one product term which is typically termed as prime implicant.
- ❖ When at least a single '1' is not covered with any other groupings but only that covers. Therefore, the prime implicant is typically termed as essential prime implicant.
- ❖ Finally, we have to note down all the prime implicants as well as essential prime implicants.
- ❖ The simplified Boolean function will be written by following all the essential prime implicants along with only the required prime implicants.

A \ B	0	1
0	0	1
1	2	3

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

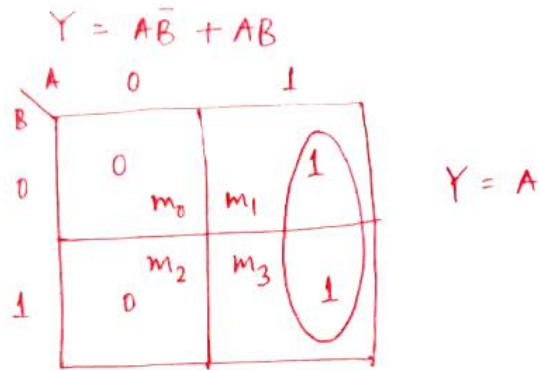
AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10



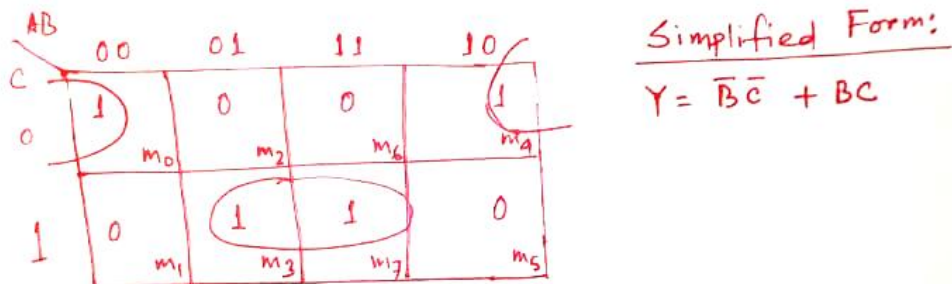
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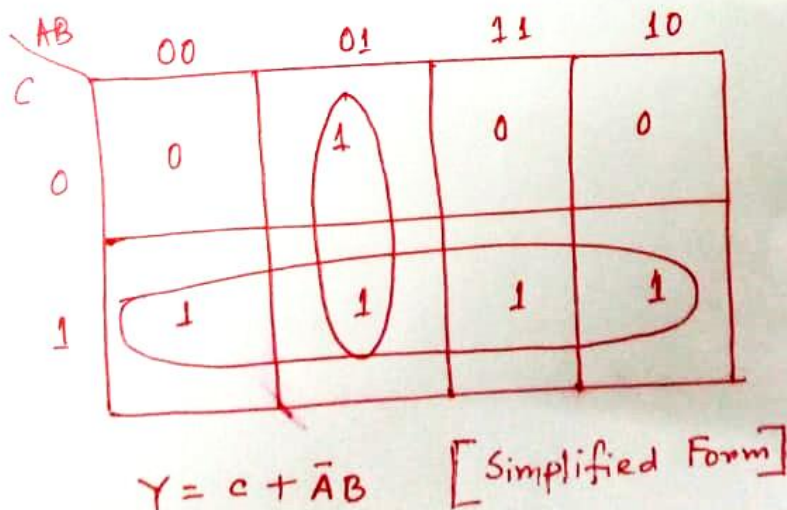
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$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC = \sum m(0, 3, 4, 7)$$



$$Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$$





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$$Y = ABCD + \bar{A}BCD + A\bar{B}CD + \bar{A}\bar{B}CD \quad [\text{SOP Form}]$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	0	0	0	0

Simplified Form:  
 $Y = CD$

Four variable K-map

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	0	0
11	1	1	1	1
10	1	1	1	1

Simplified Form:  
 $Y = C + \bar{A}BD$

Overlapping Groups

### Drawing of Digital Circuit:

Draw the digital circuit  $Y = AB + BC(B + C)$ .

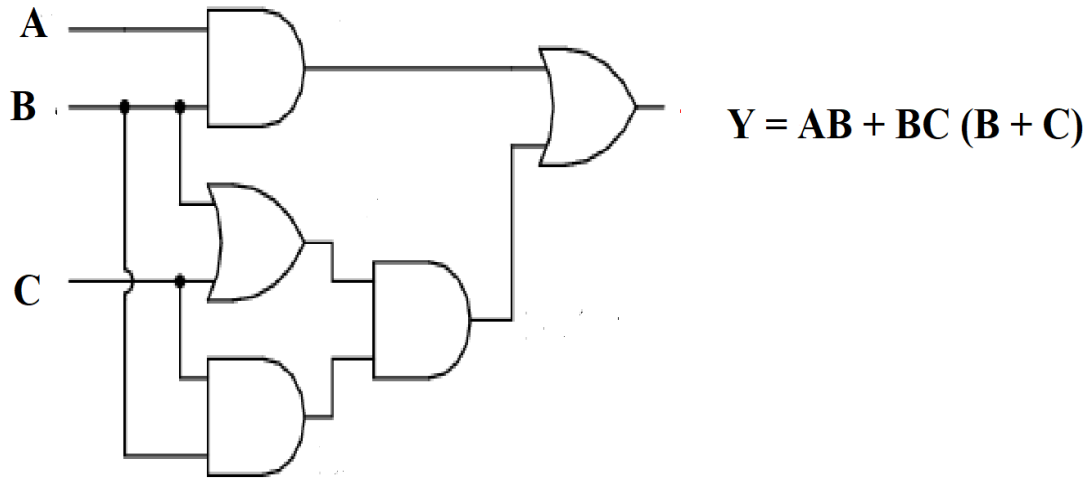
Here Y is the expression of the output.

A, B, C are the input signals.





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**Drawing of Digital Circuit**

**Frequently Asked Questions:**

1. Simplification of Boolean expression and drawing of simplified circuit.
2. Simplification of Boolean expression by K-Map and drawing of simplified circuit.
3. Drawing of circuit diagram.
4. Write down the De Morgans Law.
5. What do you mean by Max terms and mean terms?
6. What are SOP and POS?
7. Why NAND and NOR Gates are called Universal Gates?

**References:**

- (i) *Fundamentals Principles of Electronics, Author: B. Ghosh, Published by Allied Pvt. Ltd. (2018 Ed.)*
- (ii) *Electronics: Fundamentals and Applications, Author- D. Chattopaddhayay and P. C. Rakshit, Published by New Age International Publishing (2018 Ed.).*
- (iii) *Electronic Principles- Author- A. Malvino, D. J. Bates, Published by McGraw Hill. (7<sup>th</sup>Ed.).*
- (iv) *Modern Digital Electronics- Author- R.P. Jain, Published by McGraw Hill. (4<sup>th</sup>Ed.).*
- (v) *Principles of Electronics- Author- V.K. Meheta, R. K. Mehata Published by S. Chand. (11<sup>th</sup>Ed.).*



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(vi) <http://www.infocobuild.com/education/audio-video-courses/electronics/DigitalCircuits-IIT-Kharagpur/lecture-08.html>

**Link to Audio visual Lectures (e-Lectures) and NPTEL lectures on this topic given by Distinguish Professors of Indian & Foreign Universities:**

- (1) <https://nptel.ac.in/courses/117/106/117106086/>
- (2) <https://nptel.ac.in/courses/108/105/108105113/>
- (3) <https://nptel.ac.in/courses/106/105/106105185/>
- (4) <https://nptel.ac.in/courses/108/105/108105132/>
- (5) [https://onlinecourses.nptel.ac.in/noc20\\_ee70/preview](https://onlinecourses.nptel.ac.in/noc20_ee70/preview)
- (6) <https://www.youtube.com/watch?v=sUutDs7FFeA>
- (7) [https://www.youtube.com/watch?v=K73N9ES\\_8nI](https://www.youtube.com/watch?v=K73N9ES_8nI)
- (8) <https://www.youtube.com/watch?v=7UiOuA2kJu8>
- (9) <https://www.youtube.com/watch?v=0Dx7r0PFyUM>
- (10) <http://www.infocobuild.com/education/audio-video-courses/electronics/DigitalCircuits-IIT-Kharagpur/lecture-08.html>