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Paper- C1T, Sem- I

Topic- Classification of conics using the discriminant.

Classification of conics using the discriminant

Classifing a conic from its equation

The equation of any conic can be written in the form
$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
which is called a general second-degree equation in x and y. The expression $B^2 - 4AC$ is called the discriminant of the equation and can be used to determine which type of conic the equation represents.

Classifying conics

If the graph of $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is a conic, then the type of conic can be determined as follows.

Discriminant	Type of conic
$B^2 - 4AC < 0$, $B = 0$, and $A = C$	Circle
$B^2 - 4AC < 0$ and either $B \neq 0$ or $A \neq C$	Ellipse
$B^2 - 4AC = 0$	Parabola
$B^2 - 4AC > 0$	Hyperbola

If $B = 0$, each axis of the conic is horizontal or vertical. If $B \neq 0$, the axes are neither horizontal nor vertical.

(1) Classify the conic given by $2x^2 + y^2 - 4x - 4 = 0$ —(1)

Comparing (1) with the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

$\therefore \quad A = 2, B = 0, C = 1$

The value of the discriminant is $B^2 - 4AC$
$= 0^2 - 4 \cdot 2 \cdot 1 = -8$

$\therefore \quad B^2 - 4AC < 0$ and $A \neq C$

The given conic represents an ellipse.

$2x^2+y^2-4x-4=0$

$\Rightarrow 2(x^2-2x+1^2-1)+y^2=4$

$\Rightarrow 2(x-1)^2+y^2=4+2$

$\Rightarrow 2(x-1)^2+y^2=6.$

$\Rightarrow \frac{(x-1)^2}{3}+\frac{y^2}{6}=1$

By comparing this equation to $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

where $h=1$, $k=0$, $a=\sqrt{6}$ and $b=\sqrt{3}$.

(2) Classify the conic given by $4x^2-9y^2+32x-144y-548=0$. — (1)

Comparing (1) with ~~the~~ $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$.

$\therefore A=4,\ B=0,\ C=-9.$

the value of the discriminant is B^2-4AC

$= 0^2-4\cdot4\cdot(-9)$

$= 144$

$\therefore B^2-4AC>0$

the given equation represents a hyperbola.

$4x^2-9y^2+32x-144y-548=0.$

$\Rightarrow 4(x^2+8x)-9(y^2+16y)=548$

$\Rightarrow 4(x^2+8x+16-16)-9(y^2+16y+64-64)=548$

$\Rightarrow 4(x+4)^2-64-9(y+8)^2+576=548$

$\Rightarrow 4(x+4)^2-9(y+8)^2=36$

$\Rightarrow \frac{(x+4)^2}{9}-\frac{(y+8)^2}{4}=1$

$\Rightarrow \frac{(x+4)^2}{3^2}-\frac{(y+8)^2}{2^2}=1$

By comparing this equation to $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

where $h=-4$, $k=-8$, $a=3$ and $b=2$.

③ Classify the conic given by $y^2 - 72x - 450 = 0$ —— (1)

Comparing (1) with $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

$\therefore \quad A = 0, \quad B = 0, \quad C = 1$

The value of the discriminant is $B^2 - 4AC$

$= 0 - 0 = 0.$

The given conic (1) represents a parabola.

$y^2 - 72x - 450 = 0.$

$\Rightarrow \quad y^2 = 72x + 450$

$\Rightarrow \quad y^2 = 72\left(x + \frac{450}{72}\right)$

$\Rightarrow \quad y^2 = 72\left(x + \frac{25}{4}\right)$

④ Classify the conic given by $16y^2 - x^2 + 2x + 64y + 63 = 0$ —— (1)

Comparing (1) with $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

$\therefore \quad A = -1, \quad B = 0, \quad C = 16$

The value of the discriminant is $B^2 - 4AC$

$= 0 - 4 \cdot (-1) \cdot 16$

$= 64 > 0.$

The given conic (1) represents a hyperbola.

$16y^2 - x^2 + 2x + 64y + 63 = 0.$

$\Rightarrow \quad 16(y^2 + 4y) - (x^2 - 2x) + 63 = 0.$

$\Rightarrow \quad 16(y^2 + 4y + 4 - 4) - (x^2 - 2x + 1 - 1) + 63 = 0$

$\Rightarrow \quad 16(y+2)^2 - 64 - (x-1)^2 + 1 + 63 = 0.$

$\Rightarrow \quad 16(y+2)^2 - (x-1)^2 = 6$

$\Rightarrow \quad \frac{16(y+2)^2}{6} - \frac{(x-1)^2}{6} = 1.$

(5) Classify the given conic $x^2 - 4x + 16y + 17 = 0$ —— (1)

Comparing (1) with $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

$\therefore \quad A = 1, \quad B = 0, \quad C = 0$

The value of the discriminant $= B^2 - 4AC$

$= 0.$

The given conic represents a parabola.

$x^2 - 4x + 16y + 17 = 0.$

$\Rightarrow \quad (x^2 - 2x + 2^2 - 4) + 16y + 17 = 0.$

$\Rightarrow \quad (x-2)^2 + 16y + 13 = 0.$

$\Rightarrow \quad (x-2)^2 = -16y - 13$

$\Rightarrow \quad (x-2)^2 = -16\left(y + \frac{13}{16}\right)$

(6) Classify the given conic $16x^2 + 9y^2 + 24x - 36y + 23 = 0$. —— (1)

Comparing (1) with $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

$\therefore \quad A = 16, \quad B = 0, \quad C = 9.$

$\therefore \quad B^2 - 4AC = 0 - 4 \cdot 16 \cdot 9 = -144 < 0.$

$\therefore \quad B^2 - 4AC < 0$ and $A \neq C$.

The given conic (1) represents an ellipse

$16x^2 + 9y^2 + 24x - 36y + 23 = 0.$

$\Rightarrow \quad 16\left(x^2 + \frac{24}{16}x\right) + 9(y^2 - 4y) + 23 = 0.$

$\Rightarrow \quad 16\left(x^2 + \frac{3}{2}x\right) + 9(y^2 - 4y) + 23 = 0.$

$\Rightarrow \quad 16\left(x^2 + 2x\frac{3}{4} + \frac{9}{16} - \frac{9}{16}\right) + 9(y^2 - 2 \cdot y \cdot 2 + 4 - 4) + 23 = 0.$

$\Rightarrow \quad 16\left(x + \frac{3}{4}\right)^2 - \frac{144}{16} + 9(y-2)^2 - 36 + 23 = 0.$

$\Rightarrow \quad 16\left(x + \frac{3}{4}\right)^2 + 9(y-2)^2 = 22$

$\Rightarrow \quad \dfrac{\left(x + \frac{3}{4}\right)^2}{11/8} + \dfrac{(y-2)^2}{22/9} = 1.$