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DSE1AT (Elements of Modern Physics)

Topic – Planck's Quantum (Part – 1)

Introduction:

Before 1900, people had seen how Maxwell, Hertz and others scientists established firmly that light is an electromagnetic wave. Electromagnetic waves must occur in constantly varying electric and magnetic fields, where there are coupled together by both electromagnetic induction. Maxwell was able to show that the speed c of electromagnetic waves in free space is given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \approx 3 \times 10^8 \text{ ms}^{-1}.$$

where ε_0 is the electric permittivity of free space and μ_0 is its magnetic permeability. This is the same as the speed of light waves. The correspondence was too great to be accidental and therefore, Maxwell concluded that light consists of electromagnetic waves.

Interference, diffraction and polarization further demonstrated this wave nature of light. These are found only in waves. The particles we were familiar with in those days do not behave in those ways. If light consisted of a stream of classical particles, the entire screen in Young's experiment would be dark. Moreover Maxwell's theory further tells us what kind of waves they are, which is electromagnetic. Until the end of the nineteenth century the nature of light seemed settled forever.

But when we look more closely at the emission, absorption, and scattering of electromagnetic radiation, however, we discover a completely different aspect of light. We find that the energy of an electromagnetic wave is *quantized* or discrete. It is emitted and absorbed in particle-like packages of definite energy, called *photons*. The energy of a single photon is proportional to the frequency of the radiation. We will find that light and other electromagnetic radiation exhibits wave–particle duality, which says that light acts sometimes like waves and sometimes like particles. Interference and diffraction demonstrate wave behaviour, while emission and absorption of photons demonstrate the particle behaviour. This radical reinterpretation of light will lead us to the fact that light actually has dual nature, and it's known as *wave-particle duality*.

Blackbody Radiation:

We are all familiar with the glow of a hot piece of metal, which gives off visible light whose colour varies with the temperature of the metal, going from red to yellow to white as it becomes hotter and hotter. In fact, other frequencies to which our eyes do not respond are present as well. An object need not be so hot that it is luminous for it to be radiating electromagnetic energy. Therefore, all objects radiate such energy continuously irrespective of their temperatures, though which frequencies predominate depends on the temperature. At room temperature most of the radiation is in the infrared part of the spectrum and hence it is invisible.

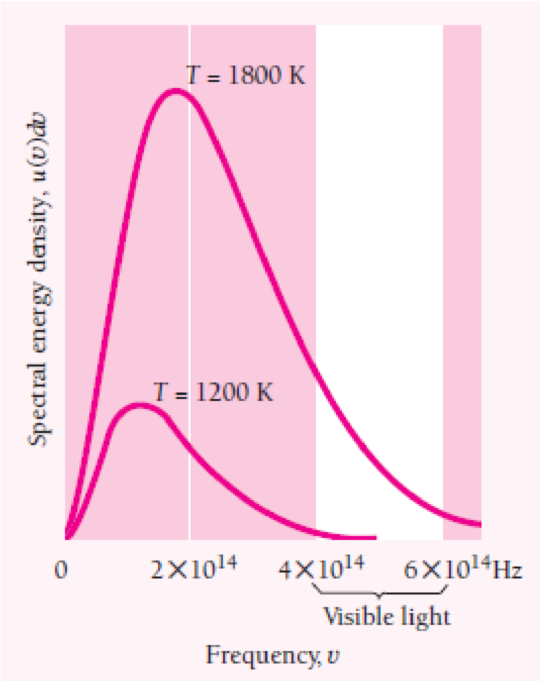

Fig. 1

The ability of a body to radiate is closely related to its ability to absorb radiation. This is to be expected, since a body at a constant temperature is in thermal equilibrium with its surroundings and must absorb energy from them at the same rate as it emits energy. It is convenient to consider as an ideal body one that absorbs all radiation incident upon it, regardless of frequency. Such a body is called a *blackbody*. We are now interested in the *blackbody radiation*. A blackbody radiates more when it is hot than when it is cold, and the spectrum of a hot blackbody has its peak at a higher frequency than the peak in the spectrum of a cooler one. We recall the behaviour of an iron bar as it is heated to progressively higher temperatures: at first it glows dull red, then bright orange-red, and eventually it becomes white hot. The spectrum (energy density $u(\nu)d\nu$ vs frequency ν) of blackbody radiation is shown in Fig. 1 for two different temperatures.

Rayleigh-Jeans Law. This spectrum was examined by Rayleigh and Jeans. They started by considering the radiation inside a cavity of absolute temperature T whose walls are perfect reflectors to be a series of standing electromagnetic waves. The number of independent standing waves in the frequency interval between ν to $\nu + d\nu$ per unit volume in the cavity turned out to be $\frac{8\pi}{c^3}\nu^2 d\nu$. Because each standing wave in a cavity originates in an oscillating electric charge in the cavity wall, two degrees of freedom are associated with the wave and it should have an average energy of $\bar{E} = 2 \times \frac{1}{2} k_B T = k_B T$, according to theorem of equipartition of energy (k_B is the Boltzmann constant). The total energy $u(\nu)d\nu$ per unit volume (or the energy density) in the cavity in the frequency interval from ν to $\nu + d\nu$ is calculated as

$$u(\nu)d\nu = \frac{8\pi k_B T}{c^3}\nu^2 d\nu$$

This is known as Rayleigh-Jeans law, which contains everything that classical theory of Physics can say about the spectrum of blackbody radiation.

Ultraviolet Catastrophe. But it was found that this formula cannot be fully correct. As the frequency ν increases towards the ultraviolet end of the spectrum, this formula predicts that the energy density should increase as ν^2, if the temperature T remains constant. In the limit of infinitely high frequencies, $u(\nu)d\nu$ therefore should also go to infinity. In reality, of course, the energy density (and radiation rate) falls to 0 as $\nu \to \infty$ (as shown in Fig. 2). This discrepancy was known as the *ultraviolet catastrophe* of classical Physics.

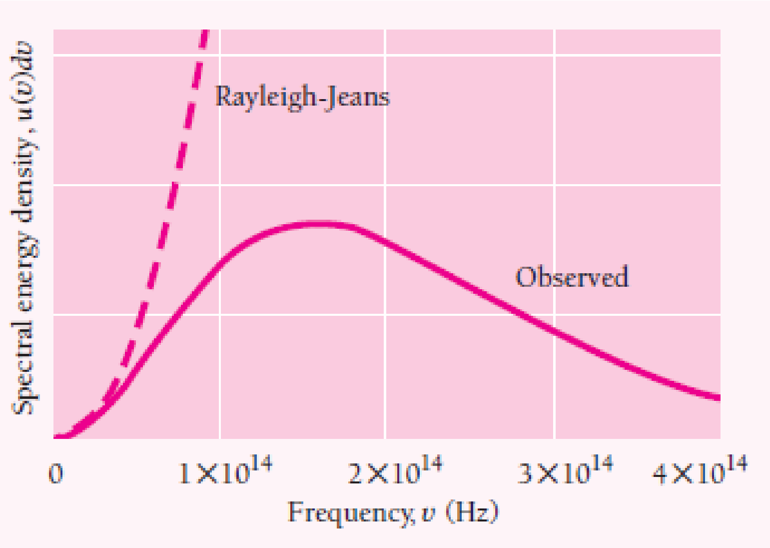

Fig. 2

Planck's Radiation Formula. In 1900, German physicist Max Planck explained the blackbody radiation by his famous quantum theory. He assumed that the oscillators in the cavity walls could not have a continuous distribution of possible energies, but must have only the specific energies given by

$$E_n = nh\nu, \text{ with } n = 0, 1, 2, \ldots$$

Here h is a universal constant, popularly called as the *Planck's constant* whose value is 6.626×10^{-34} J-s. An oscillator emits radiation of frequency ν when it drops from one energy state to the next lower one, and it jumps to the next higher state when it absorbs radiation of frequency ν. Each discrete bundle of energy $h\nu$ is called a *quantum* of the energy. With oscillator energies limited to $nh\nu$, the average energy per oscillator in the cavity walls turned out to be

$$\bar{E} = \frac{h\nu}{e^{\frac{h\nu}{k_B T}}-1}$$

Using this result, the energy density in the cavity in the frequency interval from ν to $\nu + d\nu$ is given by

$$u(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{e^{\frac{h\nu}{k_B T}}-1}$$

This is known as the Planck's radiation formula. It explains, the entire blackbody radiation spectrum, both in high and low frequency limits. At high frequencies, $h\nu \gg k_B T$ and $e^{\frac{h\nu}{k_B T}} \to \infty$. It means that $u(\nu)d\nu \to 0$ as observed, and there will be no more ultraviolet catastrophe.

At low frequencies, where the Rayleigh-Jeans law is a good approximation to the data, $h\nu \ll k_B T$ and $\frac{h\nu}{k_B T} \ll 1$. Using the approximated value of the expansion of exponential series, we get $e^{\frac{h\nu}{k_B T}} \approx 1 + \frac{h\nu}{k_B T}$ for $\frac{h\nu}{k_B T} \ll 1$. So, we get

$$\frac{1}{e^{\frac{h\nu}{k_B T}}-1} \approx \frac{1}{1+\frac{h\nu}{k_B T}-1} = \frac{k_B T}{h\nu}$$

Thus at low frequencies Planck's formula becomes

$$u(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 k_B T d\nu}{h\nu} = \frac{8\pi k_B T}{c^3}\nu^2 d\nu$$

which is the Rayleigh-Jeans law. Thus, Planck's formula turned out to be completely correct.

Photoelectric Effect:

In 1905 Planck's ideas were taken up and extended by Einstein, who showed that those explain several phenomena, the most important of which was the *photoelectric effect*. In this effect, discovered by Heinrich Hertz in 1887, a metal exposed to light is found to eject electrons from its surface. At first sight this process, which is the basis of some modern light-detecting devices, appeared perfectly consistent with classical electromagnetic theory. Light waves were known to carry energy in the form of oscillating electric and magnetic fields, and it seemed perfectly reasonable that some electrons in the metal could absorb enough of this energy to be ejected. However, closer investigation showed that several features of the process were incompatible with classical electromagnetic theory. An apparatus for investigating the photoelectric effect is shown in Fig. 3. Light is shone on one of the two electrodes in an evacuated quartz tube and electrons are ejected. If the other electrode is kept at a higher potential, it attracts these electrons, causing a current, whose magnitude indicates the number of electrons being ejected. If, instead, the second electrode is kept at a lower potential, it repels the electrons and only those electrons with enough kinetic energy to overcome the retarding potential V reach the second electrode.

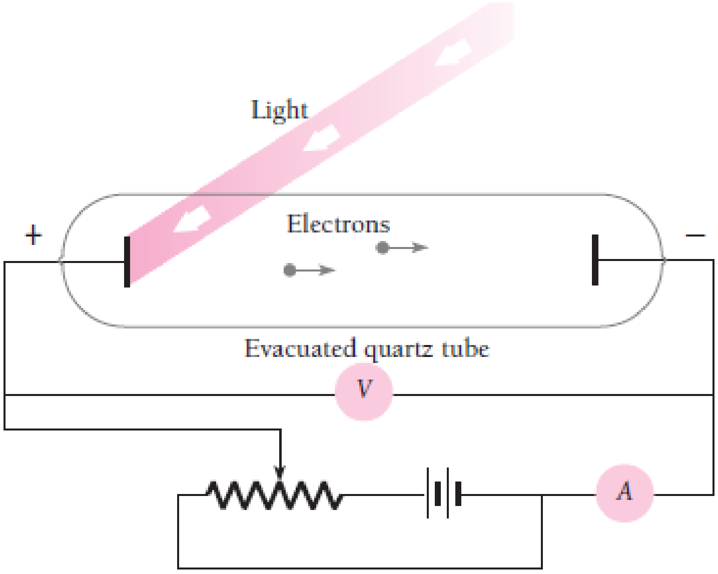

Fig. 3

As one increases the retarding potential, the current drops until at a certain *stopping potential* V_s all current ceases. Evidently V_s is given by

$$eV_s = K_{max}$$

where K_{max} denotes the maximum kinetic energy of the ejected electrons. Thus by measuring the stopping potential V_s, one can find K_{max}. When the apparatus of Fig. 3 is used to investigate the numbers and kinetic energies of the electrons ejected, two important facts emerge.

1. If the intensity of the incident light is increased, the number of ejected electrons increases (as one might expect). But quite unexpectedly, their kinetic energy does not change at all.

2. If the frequency ν of the incident light is reduced below a certain critical frequency ν_0, *no electrons are ejected* however intense the light may be.

Neither of these results is consistent with the classical view of electromagnetic waves as a continuous distribution of oscillating electric and magnetic fields. According to this view, an increased intensity means increased field strengths, which should surely eject some electrons with increased kinetic energy and there is no reasonable way to explain why low-frequency fields (however strong) should be unable to eject any electrons.

Quantum Theory of Light: Photon. Einstein proposed that as a natural extension of Planck's ideas, one should assume that the energy in a beam of light is not distributed continuously through space, but consists of a finite number of energy quanta, which are localized at points, which cannot be subdivided, and which are absorbed and emitted only as whole units. The energy of a single quantum was taken to be equal to $h\nu = h\frac{c}{\lambda}$, as suggested by Planck initially. The quantum of light was given a name, *photon*. Since it seemed unlikely that two photons would strike one electron, Einstein argued that each ejected electron must be the result of a single photon giving up its energy to the electron. With these assumptions, both of the properties mentioned above are easily explained as follows:

If the intensity of light is increased, then according to Einstein's assumptions, the number of photons is increased, but the energy $h\nu$ of an individual photon is still unchanged. With more photons, more electrons will be ejected. But since each photon has the same energy, each ejected electron will be given the same energy. Therefore, K_{max} will not change, and the 1st point is explained.

The 2nd point is explained in the following way. For any given metal, there is a definite minimum energy needed to remove an electron. This minimum energy is called the *work function* for the metal and is denoted as ϕ. If the photon energy $h\nu$ is less than ϕ, then no photons will be able to eject any electrons. In another way, if the frequency ν is less than a critical frequency ν_0 given by

$$h\nu_0 = \phi$$

no electrons will be ejected, as observed. The greater the work function of a metal, the more energy is needed for an electron to leave its surface, and the higher the critical frequency for photoelectric emission to occur.

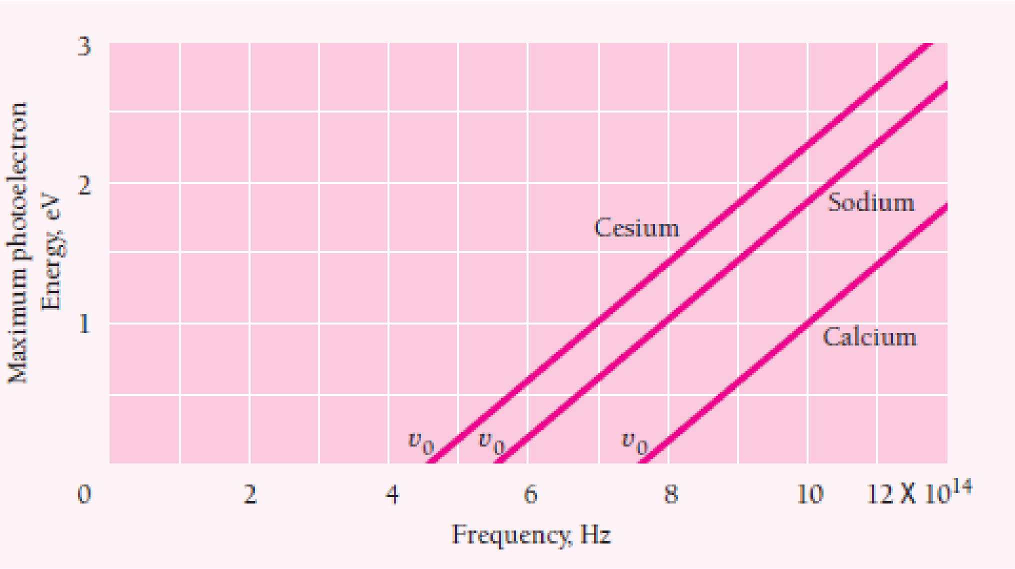

Fig. 4

Photoelectric Equation. This reasoning was carried further to make a quantitative prediction. If the frequency ν is greater than ν_0, each ejected electron should have gained energy $h\nu$ from a photon but lost ϕ or more in escaping from the metal. Thus, by conservation of energy, its kinetic energy on emerging should be $h\nu - \phi$ or less. Therefore,

$$K_{max} = h\nu - \phi$$

$$\text{or } eV_s = h\nu - \phi$$

This is Einstein's Photoelectric equation. We see that the observed maximum energy of the ejected electrons should be a linear function of the frequency of the light, and the slope of this function should be Planck's constant, h. This has been verified experimentally by Millikan (shown in Fig. 4) for several metal surfaces. It can be seen that Millikan's data are a beautiful fit to the expected straight line. In particular, by measuring the slope, Millikan was able to determine h and obtained a value in agreement with that found by Planck.

Additionally, from the x-intercept ν_0, the work functions of the metals are also experimentally calculated.

If Planck and Einstein were right that light is quantized (as they were), the question naturally arises as to why this quantization had not been observed sooner. The answer is that, by everyday standards, the energy of a single photon $h\nu$ is very small. Thus the number of photons in a normal beam of light is enormous, and the restriction of the energy to integer multiples of is correspondingly unimportant.

This concludes part 1 of this e-report.

The discussion will be continuing in the part 2 of this e-report.

Reference(s):

Concepts of Modern Physics, Arthur Beiser, McGraw-Hill

Modern Physics for Scientists and Engineers, J.R. Taylor, C.D. Zafiratos & M.A. Dubson, PHI Learning

(All the figures have been collected from the above mentioned references)