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Topic:

Elements of Modern Physics (6): Size and structure of atomic nucleus and its relation with atomic weight; Impossibility of an electron being in the nucleus as a consequence of the uncertainty principle. Nature of nuclear force, NZ graph, semi-empirical mass formula and binding energy.

Elements of Modern Physics

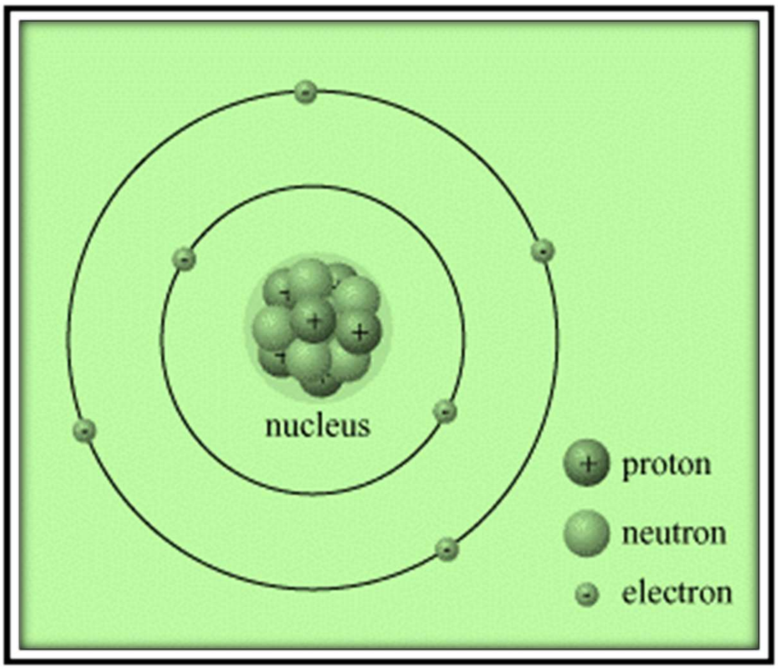
Size and Structure of Atomic Nucleus:

Nuclear Size:

Assuming the nucleus to be a sphere of radius 'R' and its volume (V) is proportional to the total number of nucleons in it or its mass number A.

Therefore, $V = \frac{4}{3}\pi R^3 \propto A$ $or,\ R \propto A^{\frac{1}{3}}$

Therefore, $R = R_0 A^{\frac{1}{3}}$, where R is the radius of the nucleus and R_0, a constant called the nuclear radius parameter. The value of R_0 ranges from (1.1 x 10^{-15} to 1.5 x 10^{-15} m or 1.1 to 1.5 fm).



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Nuclear Mass & Binding Energy:

$$M_{nuc} = M(A,Z) - Zm_e$$

Here, M_{nuc} = Nuclear Mass, $M(A,Z)$ = Atomic mass and m_e = mass of electron, A= Mass number and Z= atomic Number. Binding Energy (B.E.) =$E_B = \Delta Mc^2$, where ΔM= amount of mass disappeared.

$$\Delta M = ZM_H + NM_n - M(A,Z)$$

$$E_B = [ZM_H + NM_n - M(A,Z)]c^2$$

$$E_B = [ZM_P + NM_n + Zm_e - M_{nuc} - Zm_e]c^2$$

$$E_B = [ZM_P + NM_n - M_{nuc}]c^2$$

$$\Delta M = [ZM_P + NM_n - M_{nuc}]$$

Impossibility of an electron being in the nucleus as a consequence of the uncertainty principle

The uncertainty in position of the electron will be the same as the diameter of the nucleus $\Delta x = 2 \times 10^{-14}\ m$.

From the uncertainty principle $\Delta x . \Delta p_x \geq \hbar$

The uncertainty in momentum $\Delta p_x \geq \frac{\hbar}{\Delta x}$

The minimum uncertainty in momentum

$$\Delta p_x = \frac{\hbar}{\Delta x} = \frac{6.62 \times 10^{-34}}{2 \times 3.14 \times (2 \times 10^{-14})} = 5.278 \times 10^{-21}\ kg - m/sec$$

If, electron exist in nucleus, the minimum momentum

$$p_{min} = 5.278 \times 10^{-21}\ kg - m/sec$$

Now, for the electron,

$$E_{min}^2 = p_{min}^2 c^2 + m_0^2 c^4$$

$$E_{min}^2 = (5.278 \times 10^{-21})^2 (3 \times 10^8)^2 + (9.1 \times 10^{-31})^2 (3 \times 10^8)^4$$

$$E_{min} = 9.875\ MeV$$

If, for the electron to be present in nucleus its minimum energy should be $\sim 10\ MeV$. However, experiments shows that the maximum energy of the β-particles emitted from the radioactive nuclei is $\sim 4\ MeV$. Therefore, the electron cannot exist within the nucleus.

Nature of nuclear force

Nuclear force is the force that binds the neutrons and protons in a nucleus together. The forces inside the nucleus binds neutrons to neutrons, protons to protons and protons to neutrons are classified as strong interactions and represented by $n-n$, $n-p$, and $p-p$ forces, respectively.

The following are the few characteristics of nuclear forces: -

1. It is attractive in nature and short range force.

2. Nuclear force is identical for all nucleons and is charge independent.

3. They are strongest known force in nature.

4. They get readily saturated by the surrounding nucleons.

5. They are spin dependent.

The distances larger than 0.7 fm the force becomes attractive between spin-aligned nucleons, becoming maximal at a centre–centre distance of about 0.9 fm. Apart from this distance the force drops exponentially, until beyond about 2.0 fm separation, the force is negligible. Nucleons have a radius of about 0.8 fm.

Mass Defect and Packing Fraction

The difference between the measured atomic mass $M\ (A, Z)$ and the mass number A of a nuclide is called the mass defect, $\Delta M'$

$$\Delta M' = M\ (A, Z) - A$$

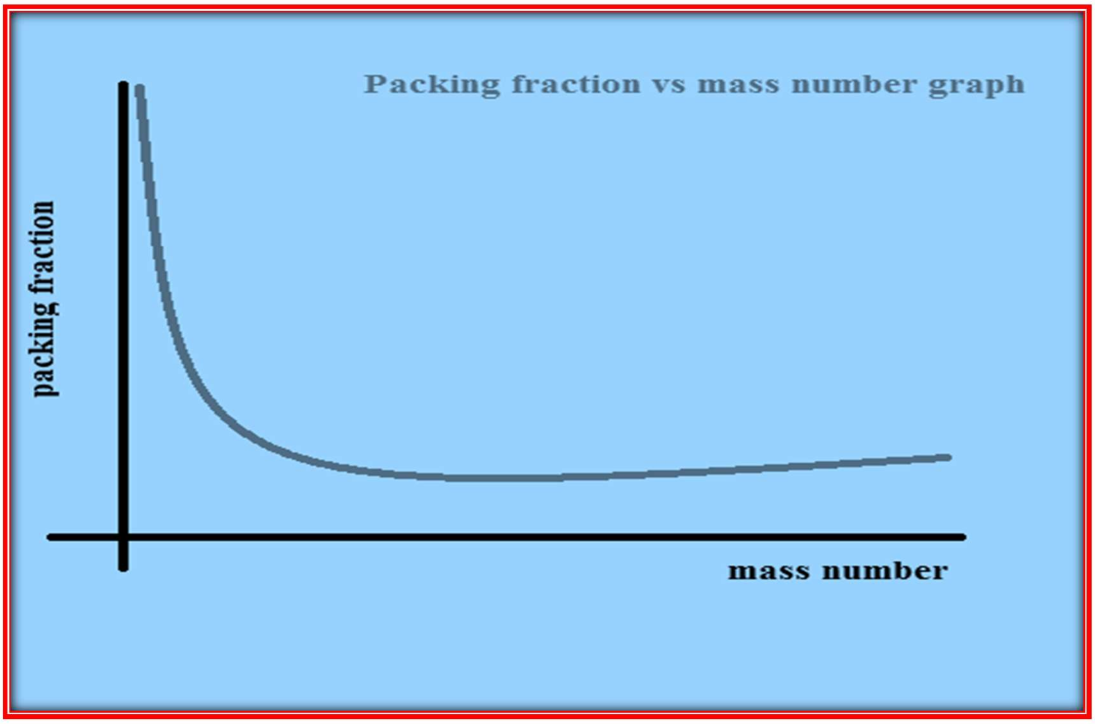
The packing fraction f is defined as the mass defect per nucleon in the nucleus,

$$f = \frac{\Delta M'}{A} = \frac{M\ (A,Z) - A}{A}$$

$$f = \frac{M\ (A,Z)}{A} - 1$$

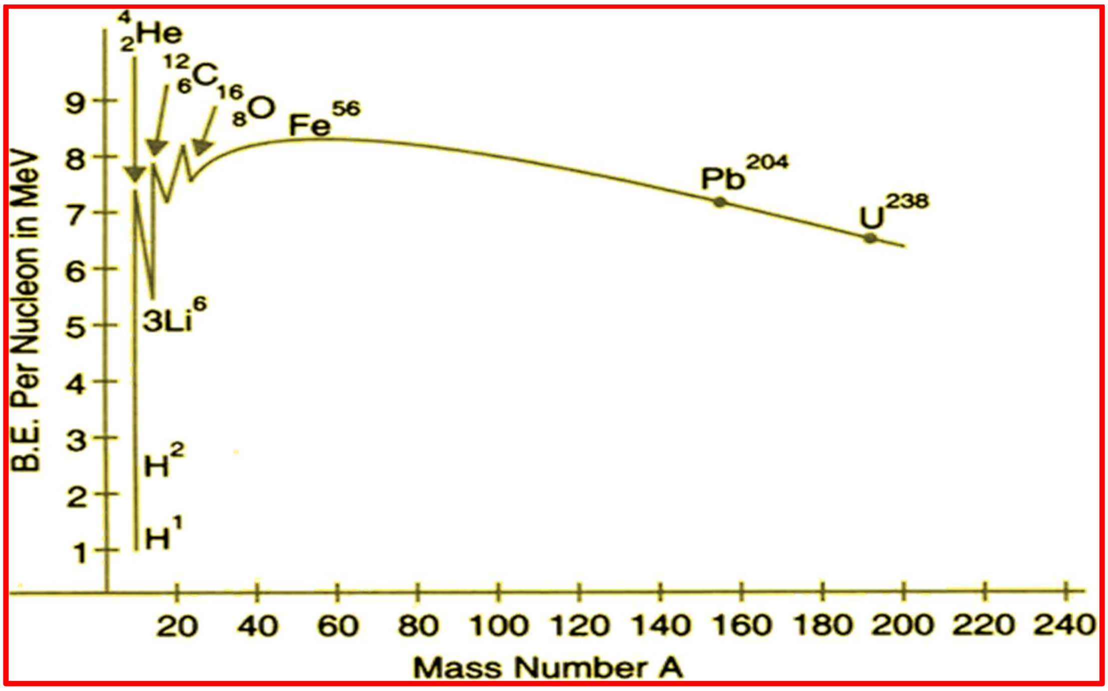
Or, $M\ (A, Z) = A(1 + f)$

And the packing fraction curve with mass number (A) becomes-



The binding fraction, $f_B = \frac{E_B}{A} = \frac{ZM_p + NM_n - M(A,Z)}{A}$ in energy unit

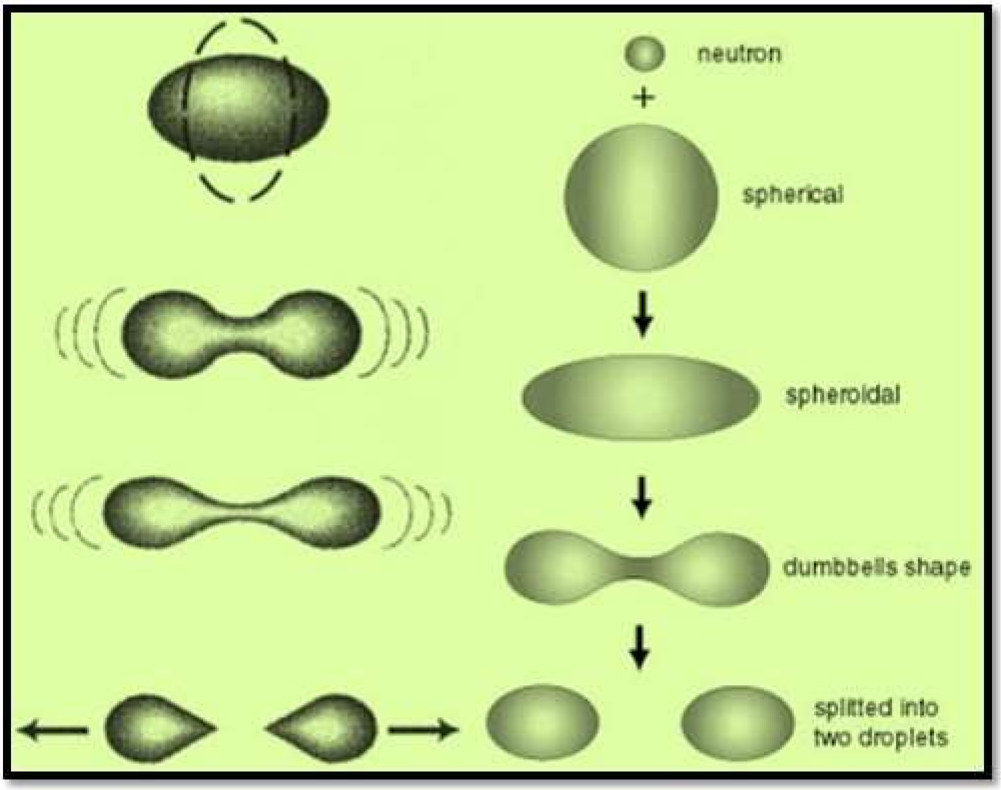
The $\frac{E_B}{A}$ $vs\ A\ curve$ becomes-



The critical survey of the curves-

1. f_B is very small for light nuclei and goes rapidly with increasing with A. For, $A = 20$ it reaches to $\sim 8\ MeV$. Then curve rises much slower and reaching at maximum value $\sim 8.7\ MeV\ for\ A = 56$. Thereafter, with increase of A, the f_B slowly decreases.
2. $Between$ 20 $< A < 180$, the variation of f_B is very small.
3. When $A > 180$, f_B decreases with increasing A and is $\sim 7.5\ MeV$ for the heaviest nuclei.
4. A rapid fluctuations $in\ f_B$ is observed for $A = 4n, where, n = 1,2,3,4$ and also for magic numbers (peaks are less prominent for $A/Z = 2,8,20,50,82,126$

Liquid Drop Model:



The similar properties between a nucleus and a liquid drop helps to explain nuclear phenomena such as the energetic of nuclear fission and the binding energy of nuclear ground levels. The nucleus has very low compressibility and well-defined surface that idea of considered the nucleus as a liquid drop. The difference as compared with liquid is that the nucleons obey Fermi statistics as well as the nucleus is a quantum fluid.

Similarities between Nucleus and Liquid Drop:

(i) The liquid drop and nucleus both possess constant density.
*(ii)*The constant binding energy per nucleon of a nucleus is similar to the latent heat of vaporization of liquid.
(iii) The evaporation of a drop corresponds to the radioactive property of nuclei.
(iv) The condensation of drops corresponds to the formation of compound nucleus.

Difference between Nucleus and Liquid Drop:

(i) There is positive charge (proton) present in the nucleus but not found in liquid drop.
*(ii)*Nucleus contains two components like proton and neutron but not in liquid drop.
(iii) The number of nucleons in the heaviest stable nucleus is much smaller than the molecules in an average liquid drop.

Semi-empirical Mass Formula:

Weizsacker in 1935 proposed the following semi-emperical formula to achieve the quantitative and basic understanding of the nuclear binding energy (B.E)

$$B.E. = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{\frac{3}{4}}}$$

Here the constant $a_v = 14.1\, MeV,\ a_s = 13\, MeV,\ a_c = 0.60\, MeV,\ a_n = 19.0\, MeV\ and$

$$\delta = 33.5\, MeV\ (for\ even - even\ or\ odd - odd\ nuclei),$$

$$\delta = 0\ (for\ even - odd\ nuclei)$$

(i) *The Volume Energy Term-*

The first term $B_v = a_v A$ represent the volume energy of all nucleons.

We know that $R \propto A^{\frac{1}{3}}$ and $R^3 \propto A$

Therefore, $V = \frac{4}{3}\pi R^3 \propto A$

Therefore, $B_v \propto A$

Hence, $B_v = a_v A$,where $a_v = volume\ coefficient$

(ii) The Surface Energy Term-

Taking nucleus to be spherical like liquid drop its surface area would be $S = 4\pi R^2$

We know that $R \propto A^{\frac{1}{3}}$ and $R^2 \propto A^{\frac{2}{3}}$

Therefore, $B_s \propto A^{\frac{2}{3}}$

Hence, $B_s = -a_s A^{\frac{2}{3}}$, where $a_s = surface\ coefficient$

(iii) The Coulomb Energy Term-

The third term, B_c is the Coulomb electrostatic repulsion between the charged particles, protons, in the nucleus. Since, each charged particle repulses all other charged particles, this term would be proportional to the $\frac{Z(Z-1)}{2}$

Therefore, the energy associated with Coulomb repulsion-

$$B_c = -k\frac{Z(Z-1)}{R}$$

We know that $R \propto A^{\frac{1}{3}}$

$B_c = -a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}}$, where $R\ is\ replaced\ by\ R_0\ A^{\frac{1}{3}}\ and\ a_s = Coulomb\ Energy\ coefficient$

(iv) The Asymmetry Energy Term-

The fourth term, B_a originates from the asymmetry between the number of protons and neutrons in the nucleus. The asymmetry energy B_a, is directly proportional to the

(a) The number of excess neutron, i.e. $(N-Z)\ or\ (A-2Z)$
(b) The fraction of nuclear volume in which the excess neutrons are present.

As the nuclear volume $\propto A$, the fractional volume of the nucleus in which excess neutrons are present will be proportional to $\frac{(N-Z)}{A}$

Therefore, $B_a \propto (N-Z)$

$$B_a \propto (N-Z)/A$$

$$B_a = -a_n \frac{(N-Z)^2}{A}$$

$$B_a = -a_n \frac{(A-2Z)^2}{A}, where\ a_n = asymmetry\ coefficient$$

(v) The Pairing Energy Term-

The pairing energy $B_a = \pm \frac{\delta}{A^{\frac{3}{4}}}$

Summing up, we get, $B.E. = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{\frac{3}{4}}}$

$$f_B = \frac{B.E}{A} = \{a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{\frac{3}{4}}}\}/A$$

Application:

I. Semi-empirical mass formula helps for prediction of stability of nuclei against β-decay.

II. The stability limit of spontaneous fission can be explained by Semi-empirical mass formula.

Frequently Asked Questions/Numerical:

For theoretical questions and problems in this section, students can solve the problems of Modern Atomic and Nuclear Physics, Author-A.B. Gupta, published by Books & Allied Pvt. Ltd. (2017 Ed.).

References:

(i) Modern Atomic and Nuclear Physics, Author-A.B.Gupta, published by Books & Allied Pvt. Ltd. (2017 Ed.).

(ii) https://en.wikipedia.org/wiki/Nuclear_binding_energy (Pictures are taken only for class note.)

Link to Audio visual Lectures (e-Lectures) on this topic given by Distinguish Professors of Indian & Foreign Universities:

(1) https://nptel.ac.in/courses/115104043/
(2) https://nptel.ac.in/courses/112/103/112103243/
(3) https://nptel.ac.in/courses/104106096/
(4) https://www.youtube.com/watch?v=6LoWEs8z1A4
(5) https://www.youtube.com/watch?v=3bwcXPmF2VA

(6)https://www.youtube.com/watch?v=8vMwzkOi0v4
(7)https://www.youtube.com/watch?v=rUU_1yUPaus
(8)https://www.youtube.com/watch?v=Rd0CJje59bE