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C12T (Solid State Physics)

Topic – Magnetic Properties of Matter (Part – 1)

Introduction:

Unlike the dielectric response, the magnetic response of most of the solids is dominated by permanent dipoles. The existence of magnetic dipoles, whether permanent or induced, can be explained only on the basis of quantum theoretical considerations. If we regard matter as made up of charged particles, there can be no magnetic moment in the state of thermal equilibrium of a strictly classical system even in the presence of an applied magnetic field. This may be argued in the following way. As the magnetic field is switched on, the circulating currents due to electrons having an associated magnetic moment are induced in the system. But these currents are destroyed by collisions in tending to bring the system finally to the state of thermal equilibrium. Thus magnetism is *essentially a quantum effect*. The two fundamental forms of magnetism, namely *diamagnetism* and *paramagnetism*, have their origin in induced and permanent magnetic moments, respectively. We can describe it in a different way. The magnetic moment of a free atom has three principal sources, namely, the spin with which electrons are attached, their orbital angular momentum about the nucleus and the change in the orbital moment induced by an applied magnetic field. The first two effects give paramagnetic contributions to the magnetization and the third gives a diamagnetic contribution.

None of the derivations of relations in classical physics including those for magnetic susceptibility is self-consistent. The permanent atomic magnetism (paramagnetism) cannot be accounted for without restricting the circulating electrons to the discrete stationary orbits as required in the Bohr's quantum theoretical model of the hydrogen atom. In the classical picture, there can be no magnetic moment associated with the current of circulating electrons because electrons in accelerated motion would radiate and finally fall on the nucleus, causing the atomic structure to collapse. Diamagnetism, where the applied magnetic field is pushed out of the system, can be appreciated similarly by realizing that the discrete quantum states occupied by electrons are stable to a certain extent only against external perturbations, like the magnetic field in the present case. In the ground $1s$ state of the hydrogen atom the orbital moment is zero, and the magnetic moment is that of the electron spin along with a small induced diamagnetic moment. In the $1s^2$ state of helium the spin and orbital moments are both zero and there is only an induced moment. Atoms with all filled electron shells have zero spin and zero orbital moment, and finite moments are associated with unfilled shells.

Linear Magnetic Materials:

We now define the fundamental physical quantities that concern the magnetic properties of materials. In vacuum or free space, the applied external magnetic field intensity ($\vec{H}$) and the magnetic induction ($\vec{B}$) are related by the following equation

$$\vec{B} = \mu_0 \vec{H}$$

where μ_0 is the magnetic permeability of free space having a constant value of $4\pi \times 10^{-7}$ SI units.

The magnetic state of a system, other than free space is specified by its magnetization $\vec{M}$, defined as the magnetic moment ($\vec{m}$) generated per unit volume of the specimen. $\vec{M}$ is related to $\vec{B}$ and $\vec{H}$ by

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

We now define linear magnetic materials as those materials, for which $\vec{M}$ is directly proportional to $\vec{H}$. Therefore, $\vec{M} \propto \vec{H}$ or $\vec{M} = \chi\vec{H}$. Here the proportionality constant χ is a material dependent quantity, and is known as the *magnetic susceptibility* of the magnetic material. Therefore we can write

$$\vec{B} = \mu_0(1 + \chi)\vec{H} = \mu\vec{H}$$

where $\mu = \mu_0(1+\chi) = \mu_0\mu_r$ is the magnetic permeability of the given magnetic material and $\mu_r = 1+\chi$ is called the *relative permeability* of the given material.

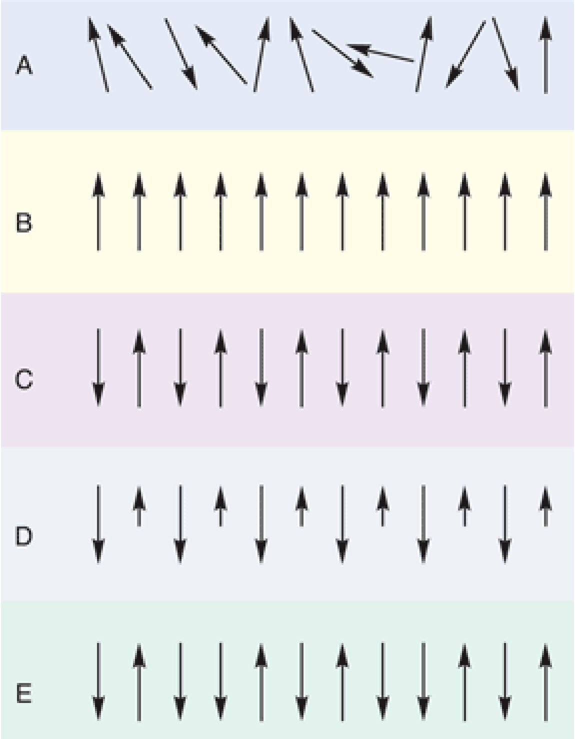

Fig. 1

Different Types of Magnetic Materials:

Magnetic properties other than diamagnetism, which is present in all substances, arise from the interactions of unpaired electrons. These properties are traditionally found in transition metals, lanthanides, and their compounds due to the unpaired d and f electrons on the metal. There are three general types of magnetic behaviours: (a) paramagnetism, in which the unpaired electrons are randomly arranged, (b) ferromagnetism, in which the unpaired electrons are all aligned, and (c) antiferromagnetism, in which the unpaired electrons line up opposite of one another. *Ferromagnetic* materials have an overall magnetic moment, whereas *antiferromagnetic* materials have a magnetic moment of zero. A compound is defined as being *ferrimagnetic* if the electron spins are orientated antiparrallel to one another but, due to an inequality in the number of spins in each orientation, there exists an overall magnetic moment. There are also enforced ferromagnetic substances (called spin-glass like) in which antiferromagnetic materials have pockets of aligned spins. Fig. 1 shows different types of magnetism and their electron spin alignments, (A) paramagnetism, (B) ferromagnetism, (C) antiferromagnetism, (D) ferrimagnetism and (E) enforced ferromagnetism.

Magnetic character of materials is typically analyzed relative to its magnetic susceptibility (χ), as shown in Table 1.

Magnetic Behaviour	Value and sign of χ
Diamagnetic	Small and negative
Paramagnetic	Small and positive
Ferromagnetic	Large and positive
Antiferromagnetic	Small and positive

Table 1

Antiferromagnetic materials can be distinguished from paramagnetic substances in that the value of χ increases with temperature, whereas χ shows decrease in value as temperature rises for paramagnetic compounds. Ferromagnetic and antiferromagnetic materials will lose magnetic character and become paramagnetic if sufficiently heated. The temperature at which this occurs is defined as the Curie temperature (T_C) for ferromagnetic compounds and the Néel temperature (T_N) for antiferromagnetic compounds.

Diamagnetism:

Diamagnetism is a small and very weak effect in many materials caused by the reaction of the rotating electrons to an applied magnetic field in accordance with Lenz's law, so that the magnetization and hence the susceptibility are both negative. Unlike paramagnetic materials, whose atoms or molecules have a net magnetic moment, the atoms or molecules of a diamagnetic material have zero magnetic moment in the absence of an external field.

Classical Langevin Theory of Diamagnetism. Let us start by considering a circular orbit of radius ρ (shown in Fig. 2) in which an electron revolves with a natural angular velocity (or frequency) ω_0 around the nucleus of charge Ze.

Then we find that the corresponding centripetal force for revolution will be provided by the Columbic force of attraction. Therefore, in the absence of any external magnetic field

$$m\rho{\omega_0}^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{\rho^2}$$

$$\text{or } \omega_0 = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 m\rho^3}}$$

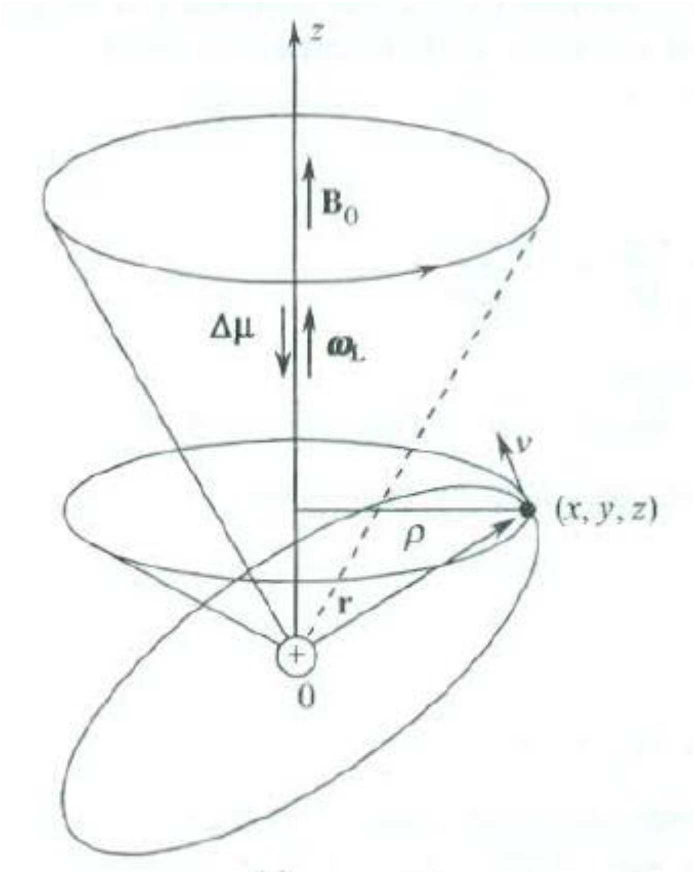

Fig. 2

The magnetic moment of the electron is given by

$$m_e = IA$$

where $I = \frac{e}{T} = \frac{e\omega_0}{2\pi}$ is the current due to the flow of the electron, $A = \pi\rho^2$ is the area of the circular orbit. So, $m_e = \frac{e\omega_0\rho^2}{2}$.

Once the magnetic field is switched on the Lorentz force acting on the electron will be $-evB = -e\omega\rho B$, where ω_0 has been changed to a new angular frequency ω. Therefore, the new equation of motion in the presence of the external magnetic field will be

$$m\rho\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{\rho^2} - e\omega\rho B$$

$$\text{or } \omega^2 = \frac{Ze^2}{4\pi\varepsilon_0 m\rho^3} - \frac{eB}{m}\omega$$

$$\text{or } \omega^2 + \frac{eB}{m}\omega - \frac{Ze^2}{4\pi\varepsilon_0 m\rho^3} = 0$$

This is a quadratic equation for ω, the roots of which are given by

$$\omega = -\frac{eB}{2m} \pm \sqrt{\frac{e^2B^2}{4m^2} + \frac{Ze^2}{4\pi\varepsilon_0 m\rho^3}}$$

$$\text{or } \omega = -\frac{eB}{2m} \pm \sqrt{{\omega_0}^2 + \frac{e^2B^2}{4m^2}}$$

$$\text{or } \omega \approx -\frac{eB}{2m} \pm \omega_0 \ (\text{if } \omega_0 \gg \frac{eB}{2m}).$$

The $\pm$ sign on ω_0 indicates that those electrons whose orbital moments are parallel to the external field will be slowed down ($\omega = \omega_0 - \frac{eB}{2m}$) and those whose moments are anti-parallel, which be speeded up ($\omega = -\omega_0 - \frac{eB}{2m}$). In both cases the change if the angular frequency will be $\Delta\omega = -\frac{eB}{2m}$. This is the outcome of so-called *Larmor theorem*. It states that in a magnetic field the motion of the electrons around a central nucleus is, to the first order in B, the same as a possible motion in the absence of B except for the superposition of a precession of the electrons with angular frequency $\omega_L = |\Delta\omega| = \frac{eB}{2m}$, where ω_L is called the *Larmor frequency*.

The precession of the electron orbit produces its own loop current and therefore, a magnetic moment (shown in Fig. 2). The average contribution of the usual motion of the electrons in their respective orbits is zero, as stated earlier. The loop current in the atomic system produced by the precessional motion is given by

$$I_L = -\frac{e\omega_L}{2\pi} = -\frac{e^2 B}{4\pi m}$$

The magnetic moment associated with the above loop current is given by

$$\Delta m_e = I_L A = -\frac{e^2 B}{4\pi m}\pi\rho^2 = -\frac{e^2 B\rho^2}{4m}$$

On summing over all electrons in the atom, the induced moment per atom becomes

$$\sum \Delta m_e = -\frac{e^2 B \sum \rho^2}{4m} = -\frac{Ze^2 B\langle\rho^2\rangle}{4m}$$

Here $\langle\rho^2\rangle = \langle x^2\rangle + \langle y^2\rangle$ is the mean square of the perpendicular distance of the electron from the field axis through the nucleus. The mean square distance of the electrons from the nucleus is $\langle r^2\rangle = \langle x^2\rangle + \langle y^2\rangle + \langle z^2\rangle$. For a spherically symmetrical distribution of charge we have $\langle x^2\rangle = \langle y^2\rangle = \langle z^2\rangle$, so that

$$\langle\rho^2\rangle = \frac{2}{3}\langle r^2\rangle$$

Therefore, we write

$$\sum \Delta m_e = -\frac{Ze^2 B\langle\rho^2\rangle}{4m} = -\frac{Ze^2 B\langle r^2\rangle}{6m}$$

If N is the number of atoms per unit volume of the system, we get the expression for magnetization

$$M = -\frac{NZe^2 B\langle r^2\rangle}{6m}$$

$$\text{or } M = -\frac{\mu_0 NZe^2 H\langle r^2\rangle}{6m} = \left(-\frac{\mu_0 NZe^2\langle r^2\rangle}{6m}\right)H$$

$$\text{or } \chi = -\frac{\mu_0 NZe^2\langle r^2\rangle}{6m}$$

This is the classical Langevin result. As expected, the sign of the diamagnetic susceptibility comes out to be negative. Diamagnetism is an essential property of all substances. On account of being weak, it is masked in paramagnetic materials under the background of their positive susceptibility.

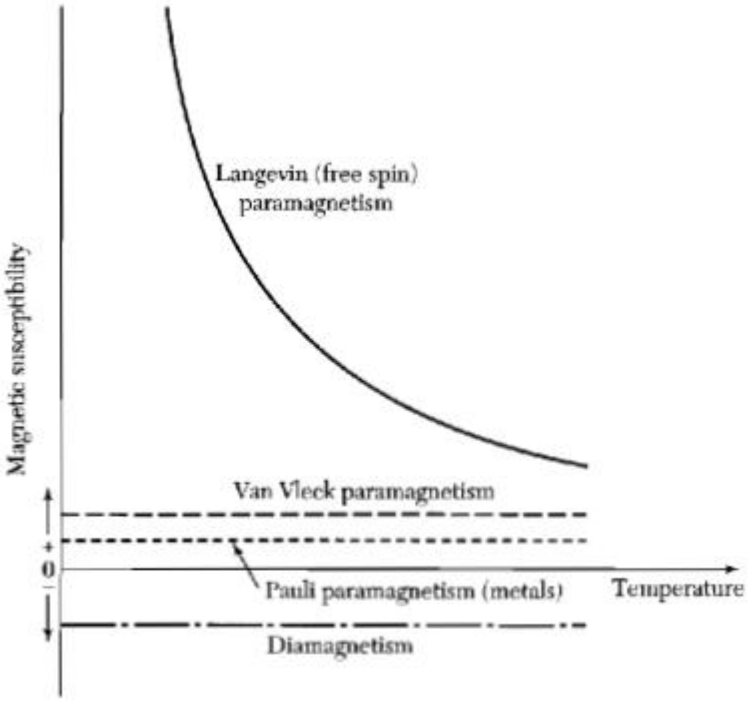

Fig. 3

In dielectric solids the diamagnetic contribution of the ion cores is described roughly by the Langevin result. But the contribution of conduction electrons in metals is more complicated. We also find that the diamagnetic susceptibility is independent of temperature, as derived from the classical result of Langevin. Fig. 3 shows the characteristic temperature dependence of magnetic susceptibilities of diamagnetic and paramagnetic substances. The diamagnetic susceptibility is essentially negative and constant with temperature.

This concludes part 1 of this e-report.

The discussion will be continuing in the part 2 of this e-report.

Reference(s):

Introduction to Solid State Physics, Charles Kittel, Wiley

Elements of Solid State Physics, J.P. Srivastava, PHI Learning

Solid State Physics, S.O. Pillai, New Age International Publishers

(All the figures have been collected from the above mentioned references)