LOTKA-VOLTERRA MODEL

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Lotka–Volterra Model:

The Lotka–Volterra predator–prey model was initially proposed by Alfred James Lotka in 1910. The Lotka-Volterra model is composed of a pair of differential equations that describe predator-prey (or herbivore-plant, or parasitoid-host) dynamics in their simplest case (one predator population, one prey population). Later, it was developed independently by Alfred Lotka and Vito Volterra in the 1920's, and is characterized by oscillations in the population size of both predator and prey, with the peak of the predator's oscillation lagging slightly behind the peak of the prey's oscillation. The model makes several simplifying assumptions:

1) the prey population will grow exponentially when the predator is absent;

2) the predator population will starve in the absence of the prey population (as opposed to switching to another type of prey);

3) predators can consume infinite quantities of prey; and

4) there is no environmental complexity (in other words, both populations are moving randomly through a homogeneous environment).

Lotka–Volterra equations:

The Lotka–Volterra equations, also known as the predator–prey equations, are a pair of first-order nonlinear differential equations, frequently used to describe the dynamics of biological systems in which two species interact, one as a predator and the other as prey.

The populations change through time according to the pair of equations:

$$\frac{dx}{dt} = \alpha x - \beta xy,$$
$$\frac{dy}{dt} = \delta xy - \gamma y,$$

where

x is the number of prey (for example, rabbits);

y is the number of some predator (for example, foxes);

$\frac{dy}{dt}$ and $\frac{dx}{dt}$ represent the instantaneous growth rates of the two populations;

t represents time;

α, β, γ, δ are positive real parameters describing the interaction of the two species.

The Lotka–Volterra system of equations is an example of a Kolmogorov model, which is a more general framework that can model the dynamics of ecological systems with predator–prey interactions, competition, disease, and mutualism.

The Lotka–Volterra model makes a number of assumptions, not necessarily realizable in nature, about the environment and evolution of the predator and prey populations:

1. The prey population finds ample food at all times.
2. The food supply of the predator population depends entirely on the size of the prey population.
3. The rate of change of population is proportional to its size.

4. During the process, the environment does not change in favour of one species, and genetic adaptation is inconsequential.
5. Predators have limitless appetite.

In this case the solution of the differential equations is deterministic and continuous. This, in turn, implies that the generations of both the predator and prey are continually overlapping.

Prey:

When multiplied out, the prey equation becomes:

$$\frac{dx}{dt} = \alpha x - \beta xy.$$

The prey are assumed to have an unlimited food supply and to reproduce exponentially, unless subject to predation; this exponential growth is represented in the equation above by the term αx. The rate of predation upon the prey is assumed to be proportional to the rate at which the predators and the prey meet, this is represented above by βxy. If either x or y is zero, then there can be no predation.

With these two terms the equation above can be interpreted as follows: the rate of change of the prey's population is given by its own growth rate minus the rate at which it is preyed upon.

Predators:

The predator equation becomes:

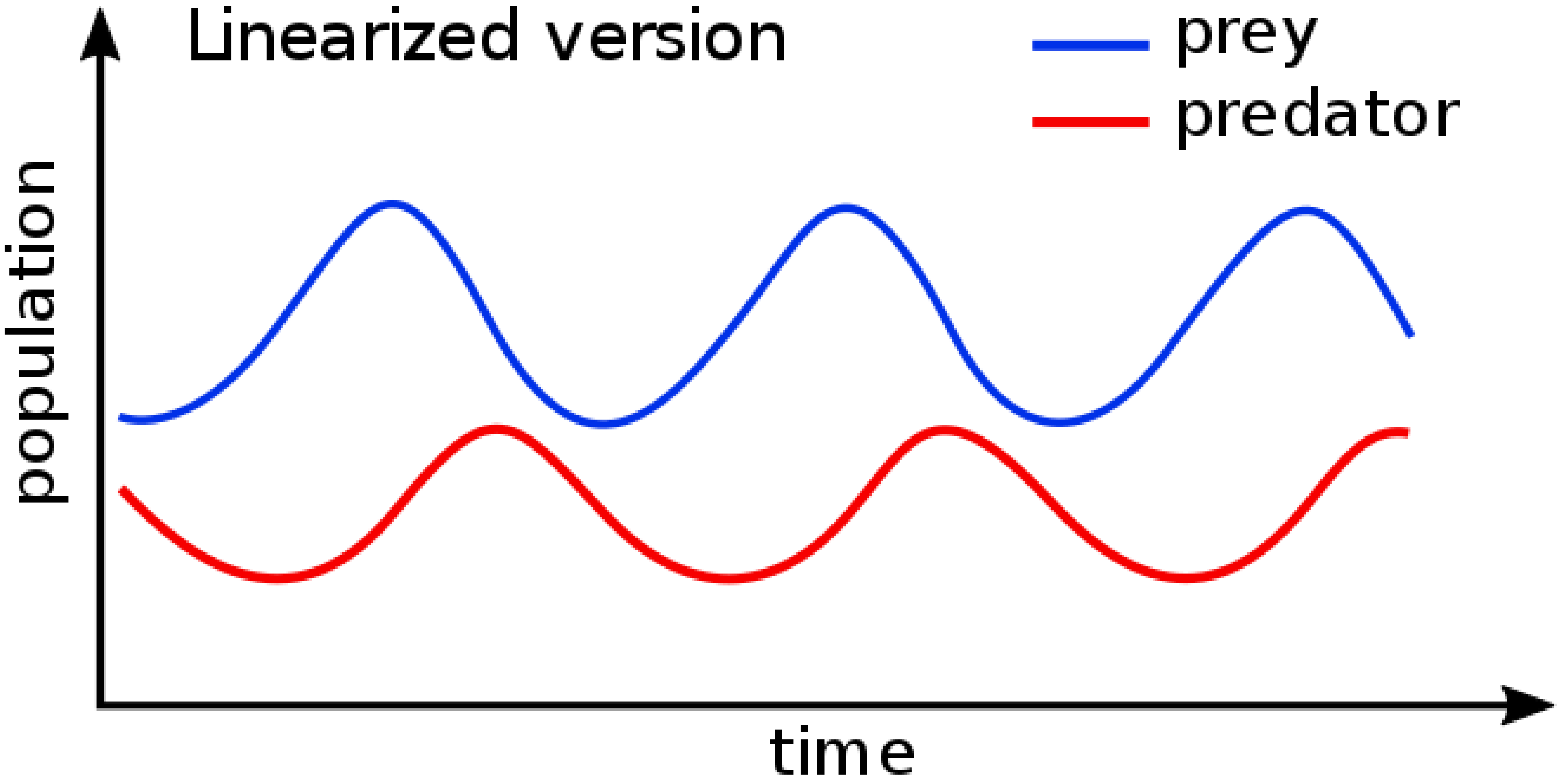
$$\frac{dy}{dt} = \delta xy - \gamma y.$$

In this equation, δxy represents the growth of the predator population. The term γy represents the loss rate of the predators due to either natural death or emigration, it leads to an exponential decay in the absence of prey.

Hence the equation expresses that the rate of change of the predator's population depends upon the rate at which it consumes prey, minus its intrinsic death rate.

Lotka–Volterra equations Graph:

If none of the non-negative parameters α, β, γ, δ vanishes, three can be absorbed into the normalization of variables to leave only one parameter: since the first equation is homogeneous in x, and the second one in y, the parameters β/α and δ/γ are absorbable in the normalizations of y and x respectively, and γ into the normalization of t, so that only α/γ remains arbitrary. It is the only parameter affecting the nature of the solutions.


Linearized version
prey
predator
population
time

Competitive Lotka–Volterra equations:

The competitive Lotka–Volterra equations are a simple model of the population dynamics of species competing for some common resource. They can be further generalised to include trophic interactions.

This is similar to the Lotka–Volterra equations for predation in that the equation for each species has one term for self-interaction and one term for the interaction with other species. In the equations for predation, the base population model is exponential. For the competition equations, the logistic equation is the basis.

The logistic population model, when used by ecologists often takes the following form:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right)$$

Here x is the size of the population at a given time, r is inherent per-capita growth rate, and K is the carrying capacity.

Two species:

Given two populations, x_1 and x_2, with logistic dynamics, the Lotka–Volterra formulation adds an additional term to account for the species' interactions. Thus the competitive Lotka–Volterra equations are:

$$\frac{dx_1}{dt} = r_1 x_1 \left(1 - \left(\frac{x_1 + \alpha_{12} x_2}{K_1}\right)\right)$$

$$\frac{dx_2}{dt} = r_2 x_2 \left(1 - \left(\frac{x_2 + \alpha_{21} x_1}{K_2}\right)\right)$$

Here, α_{12} represents the effect species 2 has on the population of species 1 and α_{21} represents the effect species 1 has on the population of species 2. These values do not have to be equal.

Because this is the competitive version of the model, all interactions must be harmful (competition) and therefore all α-values are positive. Also, note that each species can have its own growth rate and carrying capacity. A complete classification of this dynamics, even for all sign patterns of above coefficients, is available, which is based upon equivalence to the 3-type replicator equation.

N species:

This model can be generalized to any number of species competing against each other. One can think of the populations and growth rates as vectors, α's as a matrix. Then the equation for any species i becomes:

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \frac{\sum_{j=1}^{N} \alpha_{ij} x_j}{K_i} \right)$$

or, if the carrying capacity is pulled into the interaction matrix.

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \sum_{j=1}^{N} \alpha_{ij} x_j\right)$$

where N is the total number of interacting species. For simplicity all self-interacting terms αii are often set to 1.

Use of Lotka–Volterra Model:

The Lotka–Volterra model is frequently used to describe the dynamics of ecological systems in which two species interact, one a predator and one its prey.

THANK YOU