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DSE2T (Nuclear and Particle Physics)

Topic – Particle Physics (Part – 4)

We have already discussed part 3 of this e-report.

Now let us continue part 4 of it.

Discrete Symmetries:

Any set of transformations either involving space-time or some internal space - can be best understood when described in terms of a change in the reference frame. Continuous as well as discrete transformations can be discussed within this kind of framework. We have dealt with continuous symmetries in the previous e-report, and we now take our attention to discrete symmetries.

Parity Symmetry. Parity is a transformation that takes us from a *left handed* coordinate frame to a *right handed* one or vice versa. Under this transformation, which we denote by the symbol P, the space-time four-vector changes as follows

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \xrightarrow{P} \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{pmatrix} -x \\ -y \\ -z \\ t \end{pmatrix}$$

It is important to realize that the parity operation is distinct from spatial rotations because a left handed coordinate system cannot be obtained from a right handed one through any combination of rotations. In fact, rotations define a set of continuous transformations, whereas the inversion of space coordinates does not. It is clear therefore that the quantum numbers corresponding to rotations and parity are different from each other.

Classically, the components of position and momentum vectors change sign under inversion of coordinates, while their magnitudes are preserved. For example, we can write

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$$\vec{r} \xrightarrow{P} -\vec{r} \text{ and } \vec{p} = m\dot{\vec{r}} \xrightarrow{P} -m\dot{\vec{r}} = -\vec{p}, \text{ but}$$

$$r = \sqrt{\vec{r}.\vec{r}} \xrightarrow{P} \sqrt{(-\vec{r}).(-\vec{r})} = r \text{ and } p = \sqrt{\vec{p}.\vec{p}} \xrightarrow{P} \sqrt{(-\vec{p}).(-\vec{p})} = p$$

This defines the behaviour of normal scalar and vector quantities under space inversion. An important property of the parity operation is that two successive parity transformations leave the coordinate system unchanged, for example

$$\vec{r} \xrightarrow{P} -\vec{r} \xrightarrow{P} \vec{r}$$

Now, if we think of P as representing the operator implementing a parity transformation, then from the previous argument we can conclude that

$$P^2|\psi\rangle = |\psi\rangle = (+1)|\psi\rangle$$

The eigenvalues of the parity operator can therefore be only ± 1. If we have a parity invariant theory, which means a theory whose Hamiltonian H is invariant under inversion of coordinates or parity transformation, then P will have to commute with H. That means $[P, H] = 0$. And if P and H commute, both of these operators will have simultaneous eigen vectors. Therefore, the eigenstates of the Hamiltonian (H) are also eigenstates of P, with eigenvalues of either $+1$ or -1. Because a wave function transforms under P as $\psi(\vec{r}) \xrightarrow{P} \psi(-\vec{r})$, therefore the stationary (only space dependent) states of any Hamiltonian invariant under a parity transformation have definite parity, and can be classified as either even function or odd function.

We will take an example here. Let us consider the one-dimensional harmonic oscillator, whose Hamiltonian is parity invariant, since

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 \xrightarrow{P} H' = \frac{(-p)^2}{2m} + \frac{1}{2}m\omega^2(-x)^2 = H$$

And we know, the energy eigenstates of the oscillator are Hermite polynomials, which are either even or odd functions of x, but never a mixture of odd and even functions.

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Out next example will be about a rotationally invariant system in three dimension. It is obvious that, the energy eigenstates in this case are also eigenstates of the angular momentum operator, which is the conserved quantity. The wave function for the system can be written as

$$\psi_{nlm}(\vec{r}) = \psi_{nlm}(r, \theta, \phi) = R_{nl}(r) Y_{lm}(\theta, \phi)$$

where $R_{nl}(r)$ are the radial part of the wave function and $Y_{lm}(\theta, \phi)$ are the spherical harmonics, as seen in Hydrogen atom problem in quantum mechanics. The parity transformation in spherical polar coordinates takes the form

$$r \xrightarrow{P} r, \theta \xrightarrow{P} \pi - \theta \text{ and } \phi \xrightarrow{P} \pi + \phi$$

Under this transformation the radial parts and the spherical harmonics change as

$$R_{nl}(r) \xrightarrow{P} R_{nl}(-r) = R_{nl}(r)$$

$$Y_{lm}(\theta, \phi) \xrightarrow{P} Y_{lm}(\pi - \theta, \pi + \phi) = (-1)^l Y_{lm}(\theta, \phi)$$

Consequently, parity transforms any wave function that is an eigenstate of orbital angular momentum as

$$\psi_{nlm}(\vec{r}) \xrightarrow{P} (-1)^l \psi_{nlm}(\vec{r})$$

In general, a quantum mechanical wave function can also have an *intrinsic parity* or phase that is independent of its spatial transformation property of the previous equation and correspondingly, a general quantum state that is described by eigen functions of orbital angular momentum will transform under parity as

$$\psi_{nlm}(\vec{r}) \xrightarrow{P} \eta_\psi (-1)^l \psi_{nlm}(\vec{r})$$

where η_ψ is the intrinsic parity of the quantum state. We can think of the intrinsic parity as the phase analog of intrinsic spin, which when added to the orbital angular momentum yields the total angular momentum of a system. We, therefore can define the total parity of such quantum mechanical system as $\eta_{tot} = (-1)^l \eta_\psi$. We also see that the intrinsic parity satisfies the condition

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${\eta_\psi}^2 = 1$. From a detailed analysis of relativistic quantum theories we get to know that bosons have the same intrinsic parities as their antiparticles, whereas the relative intrinsic parity of fermions and their antiparticles is odd (opposite).

Conservation of Parity. When parity is a good symmetry, then the intrinsic parities of different particles can be determined by analyzing different decay or production processes. These will be demonstrated in an example given later. It should be recognized, however, that it is not possible to determine an absolute parity of any system. Because starting with some set of assignments, we can invert the parities of all states without observing a physical consequence of that change. This is similar, for example, to defining the absolute sign of electric charge or other quantum numbers. A convention is needed to define intrinsic parities of objects that differ in some fundamental way - either through their electric charge, strangeness or other characteristics. The accepted convention is to choose the intrinsic parities of the proton, the neutron and the Λ hyperon as $+1$. The parities of other particles relative to these assignments can be obtained through the analysis of parity-conserving interactions involving such particles.

When parity is conserved, it then restricts the kind of decay processes that can take place. Let us consider, for example, particle *A* decaying in its rest frame into particles *B* and *C* as given by

$$A \to B + C$$

If J denotes the spin of the decaying particle *A*, then conservation of angular momentum requires that the total angular momentum of the final state should also be J. In particular, if the two decay products are spinless, then their relative orbital angular momentum (l) must equal the spin of *A*, means $l = J$. Conservation of parity in this particular decay then implies that

$$\eta_A = (-1)^l \eta_B \eta_C = (-1)^J \eta_B \eta_C$$

Here we follow the standard convention for labelling a spin (J, which can be 0, for example) and intrinsic parity (η, which can be $+$ or $-$) of a particle written as J^η. By using the convention explained before, we find that the allowed decays correspond to

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$$0^+ \rightarrow 0^+ + 0^+$$

$$0^+ \rightarrow 0^- + 0^-$$

$$0^- \rightarrow 0^+ + 0^-$$

It also follows that certain decays are forbidden because they violate parity conservation, for example

$$0^+ \nrightarrow 0^+ + 0^-$$

$$0^- \nrightarrow 0^+ + 0^+$$

$$0^- \nrightarrow 0^- + 0^-$$

Parity of π^- Meson. Here we will take an example of the parity of π^- meson. Let us Consider the absorption of very low-energy π^- mesons on deuterium nuclei given as

$$\pi^- + d \rightarrow n + n$$

to produce two neutrons. If l_i and l_f denote the orbital angular momenta in the initial and final states, respectively, then conservation of parity in the previous reaction would require

$$(-1)^{l_i}\eta_\pi\eta_d = (-1)^{l_f}\eta_n\eta_n = (-1)^{l_f}{\eta_n}^2$$

where η_π, $\eta_d$and η_n represent intrinsic parities of the three particles. Because the intrinsic parity of the deuterium is $\eta_d = +1$, and ${\eta_n}^2 = +1$, it follows that

$$(-1)^{l_i}\eta_\pi = (-1)^{l_f}$$

$$\text{or } \eta_\pi = (-1)^{l_f - l_i}$$

The capture process is known to proceed from an $l_i = 0$ state, and consequently, we get that

$$\eta_\pi = (-1)^{l_f}$$

This is how the parity of π^- meson is determined.

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The Standard Model: Quarks:

A series of measurements performed in the late 1960s at the Stanford Linear Accelerator Centre (SLAC) on electron scattering from hydrogen and deuterium revealed that the data could be most easily understood if protons and neutrons were composed of point-like objects that had charges of $+\frac{2}{3}e$ and $-\frac{1}{3}e$. These experiments, led by Jerome Friedman, Henry Kendall and Richard Taylor, corresponded to a modern parallel of the original work of the Rutherford group, where, instead of finding point-like nuclei within atoms, the presence of point-like *quarks* or *partons* was deduced from the characteristics of inelastically scattered electrons.

It is perhaps worth expanding somewhat on the difference between elastic and inelastic scattering of electrons from nucleon targets. For elastic scattering at high energy, the form factor obtained from measurements at low energies, provides an adequate description of the differential cross section. However, inelastic scattering, where the proton does not stick together, offers the possibility of probing for substructure within the nucleon. In particular, the inelastic scattering of electrons at large q^2 corresponds to interactions that take place at very small distances, and are therefore sensitive to the presence of point-like constituents within the nucleon. In fact, the form factor for inelastic scattering at large q^2 becomes essentially independent of q^2, reflecting the presence of point-like objects within the nucleon. This is reminiscent of the large-angle contribution to the Rutherford scattering of low-energy α-particles on the point-like nucleus of the atom. It was eventually clarified that the nucleon contained charged *quarks* as well as neutral *gluon*s, both described by their individual characteristic momentum distributions.

Quarks and Leptons:

As we have seen earlier, each charged lepton has its own neutrino, and there are three families (or flavours) of such leptons, namely,

$$\binom{\nu_e}{e^-}, \binom{\nu_\mu}{\mu^-} \text{ and } \binom{\nu_\tau}{\tau^-}$$

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In writing this, we have used the convention introduced previously in connection with strong isospin symmetry, namely, the higher member of a given multiplet carries a higher electric charge.

The quark constituents of hadrons also come in three families, shown as

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix} \text{ and } \begin{pmatrix} t \\ b \end{pmatrix}$$

where we come across six different types of quarks, namely up (u), down (d), charm (c), strange (s), top (t) and bottom (b). The charges and several new quantum numbers of the different quarks were given in Table 1. The baryon numbers are $B = \frac{1}{3}$ for all the quarks.

Quark with Symbol	Electric Charge (e)	Flavour Quantum Numbers			
		Strangeness	Charm	Bottomness	Topness
Up (u)	$+\frac{2}{3}$	0	0	0	0
Down (d)	$-\frac{1}{3}$	0	0	0	0
Charm (c)	$+\frac{2}{3}$	0	1	0	0
Strange (s)	$-\frac{1}{3}$	-1	0	0	0
Top (t)	$+\frac{2}{3}$	0	0	0	1
Bottom (b)	$-\frac{1}{3}$	0	0	-1	0

Table 1

Although the fractional nature of their electric charges was deduced indirectly from electron scattering only for the u and d quarks, phenomenologically, such charge assignments also provide a natural way for classifying the existing hadrons as bound states of quarks. Quarks also appear to have flavour quantum numbers, as given in Table 1. For example, because we defined the strangeness of the K^+ as $+1$, we will see that the strange quark will have to be assigned a strangeness of -1. The charm, top and the bottom quarks, correspondingly, carry their own flavour quantum numbers. Of course, each quark has its own antiquark, which has opposite electric charge and other internal quantum numbers such as strangeness and charm. The isotopic spin of quarks can be

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inferred from the generalized Gell-Mann-Nishijima relation as discussed before. Free quarks are not observed in nature. The top quark is as free as a quark can get, but it decays so rapidly that it does not have sufficient time to form hadrons, reflecting the fact that its weak interactions are stronger than its strong interactions.

This concludes part 4 of this e-report.

The discussion will be continuing in the part 5 of this e-report.

Reference(s):

Introduction to Elementary Particles, D. J. Griffiths, John Wiley & Sons

Introduction to Nuclear and Particle Physics, A. Das & T. Ferbel, World Scientific

(All the figures have been collected from the above mentioned references)