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Paper: DSE1T (Classical Dynamics)

Topic: Classical Mechanics of Point Particles (Part-1)

Brief Review of Newtonian Mechanics:

Classical Mechanics is a branch of physics which deals with the motion of ~~the~~ macroscopic objects. In classical mechanics, if the present state of an object is known then it is possible to predict the future state i.e. how the particle will move in the future (it's called determinism) and also possible to find out the past state i.e. how it moved in the past (it's called the reversibility). The mechanics based on Newton's laws of motion is known as the Newtonian mechanics It actually consists of the physical concepts and mathematical methods developed by Newton, Leibniz and others in the seventeenth century to describe the motion of objects under the influence of a system of forces. Later (in 18th and 19th centuries), more abstracts methods

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were developed which led to the reformulation of classical mechanics known as Lagrangian mechanics and Hamiltonian mechanics.

Classical Mechanics gives extremely accurate results for the objects that are not extremely massive and their speeds are small compared to the speed of light (c). When the size of the objects is about atomic dimension, the classical mechanics fails to describe the motion of such objects and we need different kinds of mechanics called quantum mechanics. Also, when the speed of object(s) is of the order of the speed of light (c) then special theory of relativity is needed to describe the motion. And when the objects are extremely massive, like stars general theory of relativity is applicable.

Let us now review Newtonian mechanics very briefly before we quickly switch on to the Lagrangian mechanics. Let's start with the three laws of motion.

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1. First law : In absence of any external force, an object continues to stay in its state of rest or of constant velocity. The property of an object that it can not change its state of rest or constant velocity is called the inertia and the influence by which object's velocity changes is called the force. First law of motion is also known as law of inertia.

2. Second law : The time-rate of change of momentum is proportional to the force applied on the object. Mathematically,

$$\frac{d\vec{P}}{dt} = \vec{F} \qquad \text{—— (1)}$$

Here we have taken the proportionality constant as 1, as the ~~definition of~~ force units (in SI system) is defined in such way that the proportionality constant becomes 1.

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From the first law of motion we have learnt that every object possesses the property of inertia which can be thought of as the resistance to motion. We also know that different objects ~~have~~ react differently to the same applied force, which means different objects have different ~~mass as~~ amount of inertia. The quantitative measure of inertia is called the mass of an object and its is denoted by m. Now, the momentum is defined by

$$\vec{P} = m\vec{v} \qquad \text{——— (2)}$$

The second law of motion can now be written as

$$\frac{d}{dt}(m\vec{v}) = \vec{F}$$

Considering that the mass does not change during the motion, we get

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a} \qquad \text{——— (3)}$$

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In words : Force = mass × acceleration

This is the most fundamental law in classical mechanics. Now, we can show that first law is the special case of second law. If there is no force acting on an object then $\vec{F} = 0$. ~~which gives~~ Now, the mathematical form of 2^{nd} law i.e eqn. ③ gives

$$\vec{a} = \frac{d\vec{v}}{dt} = 0$$

$$\Rightarrow \vec{v} = \text{constant (including zero)}.$$

This means an object moves with the constant velocity or remains in rest if it was so previously in the absence of external force. This is the first law of motion.

3. Third law : To every action there is an equal and opposite reaction. It means, if two objects (say 1 and 2) are interacting with each other then

$$\vec{F}_{12} = -\vec{F}_{21} \qquad \text{———— (4)}$$

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In words: Force acting on 1st object due to 2nd
= − Force on 2nd object due to 1st

Now, from the 2nd law we can say that

$$\vec{F}_{12} = \text{Rate of change of momentum of object 1}$$
$$= \frac{d}{dt}(m_1 \vec{v}_1) = m_1 \frac{d\vec{v}_1}{dt} = m_1 \vec{a}_1$$

Similarly, $\vec{F}_{21} = m_2 \vec{a}_2$

So, from eqn. (4), we get $m_1 \vec{a}_1 = -m_2 \vec{a}_2$

Considering only the magnitude we get

$$m_1 a_1 = m_2 a_2$$

$$\text{or,} \quad m_2 = \frac{m_1 a_1}{a_2} \quad \text{—— (5)}$$

Eqn. (5) mathematically defines the mass. By measuring the ratio of the accelerations, we can ~~measure~~ calculate the ratio of the masses. If we consider m_1 to be the mass of standard body (say 1 kg), then we can determine the mass of the 2nd body i.e. m_2 precisely.

Reference:

1. *Classical Mechanics - J. C. Upadhyaya*
2. *Classical Mechanics - N. C. Rana & P. S. Joag*