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Topic:

Superconductivity: London's Equation and Penetration Depth. Idea of BCS theory (No derivation)

London's Equation & Penetration Depth:

London's Equation

We assume that a coil of superconductor with temperature $T < T_c$, which consist normal current density J_n and super current density J_s, due to normal and super electron. Here, the normal electron scattered by lattice atoms in absence of electric field as well as J_n dies out. However, super electrons do not scattered by lattice atoms as well as J_s increases. With rise of temperature, J_s decreases and J_n increases and finally at $T > T_c$, J_s dies out.

If m = mass of electron, n_s = superconducting electron density, v_s= drift velocity, **E**= electric field

Therefore, $m\frac{dv}{dt} = -eE$

$J_s = -n_s ev$

$\frac{dJ_s}{dt} = -n_s e\frac{dv}{dt} = -n_s e\,(-\frac{eE}{m})$

$\frac{dJ_s}{dt} = -n_s = -n_s\frac{e^2}{m}E$

Therefore, $E = \lambda\,\frac{dJ_s}{dt}$

This equation is called London First equation

$\lambda = \frac{m}{n_s e^2}$ is typically termed as characteristics of superconductor

When $E = 0, \frac{dJ_s}{dt} = 0$

Therefore, $J_s = constant$

From Maxwell's third equation,

$\nabla\, X\, E = -\frac{\partial B}{\partial t}$

$\frac{\partial B}{\partial t} = -\nabla\, X \lambda J_s$

$\frac{\partial B}{\partial t} = -\lambda \nabla X \partial J_s / \partial t$

Therefore, $\frac{\partial}{\partial t} [B + \lambda \nabla X J_s] = 0$

Therefore, $B = -\lambda \nabla X J_s$

This is called London second equation.

London's Penetration Depth

From Maxwell's fourth equation,

$$\nabla X B = \mu_0 \left[J_s + \epsilon_0 \frac{\partial E}{\partial t} \right] = \mu_0 J_s$$

$$\epsilon_0 \frac{\partial E}{\partial t} = displacement\ current\ which\ is\ neglected$$

Therefore,

$$\nabla X (\nabla X B) = \mu_0 \nabla X J_s = \mu_0 (-\frac{B}{\lambda})$$

Therefore,

$$-\nabla^2 B + \nabla(\nabla . B) = \mu_0 (-\frac{B}{\lambda})$$

We all know that due to nonexistence of monopole $\nabla . B = 0$

Therefore, finally,

$$-\nabla^2 B = \mu_0 (-\frac{B}{\lambda})$$

$$\nabla^2 B = \mu_0 \frac{B}{\lambda}$$

$$\nabla^2 B = \frac{B}{\delta^2}$$

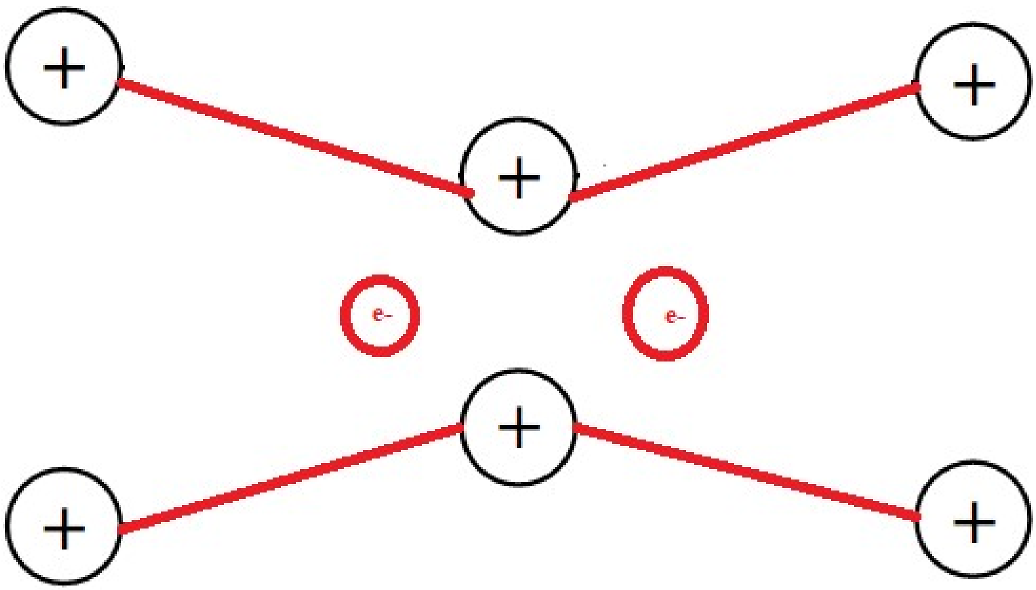
$\delta = \sqrt{\frac{\lambda}{\mu_0}}$ is called the London penetration depth of magnetic field.

The London penetration depth for a superconductor typically characterizes the distance to which a magnetic field penetrates into the superconductor along with becomes equal to $\frac{1}{e}$ times that of the magnetic field at the surface of the superconductor.

Idea of BCS Theory

The theory formulated by John Bardeen, Leon Cooper, and Robert Schrieffer. This theory elucidates on the phenomenon that at low temperature, the current of electron pairs flows in some materials/superconductors without resistance. They explained this phenomenon by the theory which is typically termed as BCS theory. They explained that the single negatively charged electron somewhat distorts the lattice of the atoms in the superconducting material. Due to this phenomena a small excess of positive charge is drawn. This excess of positive charge thereafter attracts a second electron. The attraction between the electrons is weak. This indirect as well as weak attraction binds the electrons together. This electron pair is typically termed as Cooper pair.

Cooper had discovered the Cooper pairs. The motions of all of the Cooper pairs within the superconductor are correlated. Also, the motions of all of the Cooper pairs within the superconductor constitute a co-ordination which functions like as a single object. When an electrical voltage is applied to superconductor; all the Cooper pairs are to moving state which constitute a current. The current continues to flow indefinitely in the superconductor if the voltage is detached due to the fact that the Cooper pairs come across no opposition. All of the Cooper pairs would have to be halted at the same time for the stopping of flowing of current. The Cooper pairs separate into individual electrons when the superconductor become warmed. Finally, the material becomes normal state i.e. non-superconducting state.



Cooper Pair of Electron: Superconductivity

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Frequently Asked Questions:

1. State London's Equation and penetration depth.
2. Derive London's Equation and penetration depth.
3. State the BCS theory of superconductivity?
4. What are Cooper pairs?

References:

(i) *Introduction to Solid State Physics*, ***Global Author-Charles Kittel, Published by WILEY (Global Ed.).***
(ii) *Solid State Physics (Theory, Problems and solutions)*, ***Author- Jyotirmoy Guha, Published by Books and Allied Pvt. Ltd. (2nd Ed.).***
(iii) *Solid State Physics*- ***Author- S. O. Pillai, Published by New Age International Publishers. (7th Ed.).***
(iv) *https://www.britannica.com/science/BCS-theory*

Link to Audio visual Lectures (e-Lectures) and NPTEL lectures on this topic given by Distinguish Professors of Indian & Foreign Universities:

(1) *https://www.digimat.in/nptel/courses/video/115103108/L01.html*
(2) *https://www.youtube.com/watch?v=GglT1RoBPzg*
(3) *https://www.youtube.com/watch?v=D-9M3GWoBrw*