Dr. Avradip Pradhan,
Assistant Professor,
Department of Physics,
Narajole Raj College, Narajole.

DSE2T (Nuclear and Particle Physics)

Topic – Particle Accelerators (Part – 2)

We have already discussed part 1 of this e-report.

Now let us continue part 2 of it.

Cyclotrons:

An alternative to the single-stage electrostatic accelerators is a circular device, in which a beam of particles makes many (perhaps hundreds) of cycles through the device, receiving a small voltage increment in each orbit until the particle energy reaches the MeV range. The earliest and simplest of these accelerators is known as the *Cyclotron*, also called the magnetic resonance accelerator.

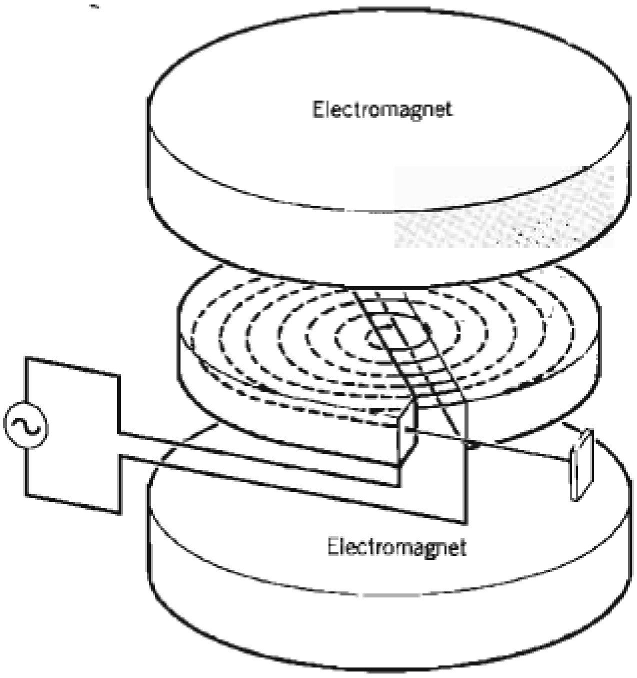

Fig. 1

The cyclotron is illustrated schematically in Fig. 1. The beam is bent into a circular path by a vertically applied magnetic field and the particles orbit inside two semicircular metal chambers called "Dees" because of their D-like shape (shown in Fig. 2). The dees are connected to a source of alternating voltage.

When the particles are inside the dees, there is no potential drop. Therefore there is no electric field and the charged particles follow a circular path under the influence of the magnetic field. Only in the gap between the dees, the particles feel the accelerating voltage and gain a small energy each cycle.

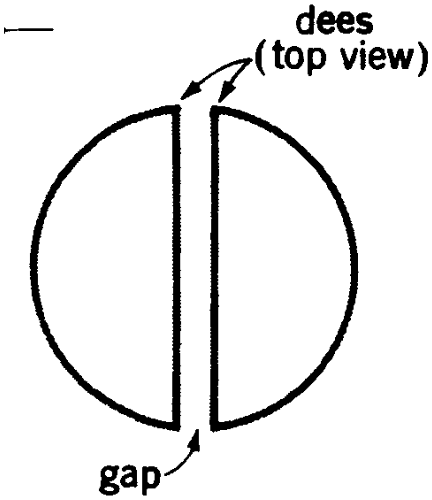

Fig. 2

The essential design idea of a cyclotron was conceived by Ernest Lawrence in the year of 1929. The critical feature is that the time it takes for a particle to travel one semicircular path is independent of the radius of the path. As particles spiral to larger radii, they also gain energy and move at a greater speed, and the gain in path length is exactly compensated by the increased speed. If the time period of the AC voltage on the dees is set equal to the circular orbital time, then the field alternates in exact synchronization with the passage of particles through the gap, and the particle sees an accelerating voltage each time it crosses the gap.

The Lorentz Force in the circular orbit qvB, provides the necessary centripetal acceleration to maintain the circular motion at an instantaneous radius r, and we can write for a charged particle of mass m

$$F = qvB = \frac{mv^2}{r}$$

Therefore,

$$r = \frac{mv}{qB}$$

The time period for the orbital motion is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

and the frequency associated with one revlution, known as *Cyclotron Frequency* is given by

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

In order to have perfect synchronization, the frequency of the AC voltage has to be the same as above and it is called as the *Resonant Frequency* for a particle of charge q and mass m moving in a uniform magnetic field B. The previous equation shows that f and B are intimately linked. For a given field strength, the frequency can only have a certain value for resonance. The velocity increases gradually as the particle spirals outward, and the greatest velocity (v_{max}) occurs at the largest radius R, when we write $v_{max} = \frac{qBR}{m}$.

Therefre, the maximum kinetic (T_{max}) energy obtained from a cyclotron can be written as

$$T_{max} = \frac{1}{2} m {v_{max}}^2 = \frac{q^2 B^2 R^2}{2m}$$

Previous result shows that it is advantageous to build cyclotrons with large fields and large radii. It is also important to note that the amplitude of the AC voltage between the dees does not appear in any of these expressions. A larger voltage means that the particle gets a larger kick with each orbit, but it makes a smaller number of orbits and emerges with the same maximum energy as it would with a smaller voltage.

There are a few limitations of the cyclotron. As the beam in a cyclotron travels outward toward the edge of the machine, the magnetic field lines are diverted somewhat from the true vertical. As a result, the field loses its uniformity and the resonance condition can no longer be maintained if the frequency is held constant. A more serious difficulty comes from the relativistic behaviour of the accelerated particles. Replacing the momentum mv with the relativistic value γmv where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$, we see that to maintain the resonance condition as v increases we must also increase B, and so the field should be larger at the larger radii. In the basic design of the fixed-field, fixed-frequency cyclotron, there is no acceptable way to compensate for the relativistic effect, and this provides an ultimate limit on the size of such machines. For protons, an energy of about 40 MeV is the maximum that can be achieved, corresponding to $\gamma = 1.04$.

Synchrotrons:

To overcome the problem of relativistic behaviour, one solution is to vary the frequency (f), resulting in a frequency modulated cyclotron, called a *synchrocyclotron*. To understand the operation of the synchrocyclotron, we must first discuss the concept of phase stability of cyclotron orbits. It is obvious that in a variable-frequency cyclotron, a continuous beam is not possible, for the time to travel the semicircular orbits will no longer be constant and equal to the half-period (which is now variable). Thus the particles travel through the cyclotron in bunches, and the frequency is swept from its maximum value (when the bunch is near the centre, the particles are only slightly accelerated, and the relativistic increase in mass is slight) to its minimum value (when the bunch is ready to exit the cyclotron, the maximum energy is attained, and the mass has its largest value). Particles in the bunches will arrive at the gap between the dees at different times. Phase stability provides a sort of time-focusing effect. Those particles that arrive early are delayed somewhat and on the next cycle are closer to the centre of the bunch, while those that arrive late are advanced and likewise pushed closer to the centre.

To see how this occurs, imagine a particle circulating at the centre of a bunch and arriving at the gap at the instant the accelerating voltage passes through zero (shown in Fig. 3). Such a particle would circulate forever in this stable orbit, called a *synchronous orbit*. Now let us suppose another particle in the bunch arrives a bit earlier, at point b of Fig. 3. This particle sees an accelerating potential, which will increase the energy and radius of the orbit, but the mass will increase, thereby decreasing the orbital frequency. Because its frequency is lower, it arrives at the next gap crossing later in phase, that is, closer to the centre of the bunch. Similarly, a particle arriving originally later than the centre of the bunch will be decelerated, and the decrease in mass increases the angular frequency and pushes the particles closer to the centre of the bunch at the next gap crossing. Particles in a bunch therefore may perform oscillations with respect to the synchronous orbit, moving first ahead, then closer to, and then perhaps behind the particles in the synchronous orbit. In the synchrocyclotron, as the frequency is slowly decreased, the radius of the synchronous orbit will increase and with it the energy will increase. With each passage of the gap, the decreasing frequency causes particles to appear at the early position with respect to the synchronous orbit. These particles are both accelerated and bunched by the phase-stability effect.

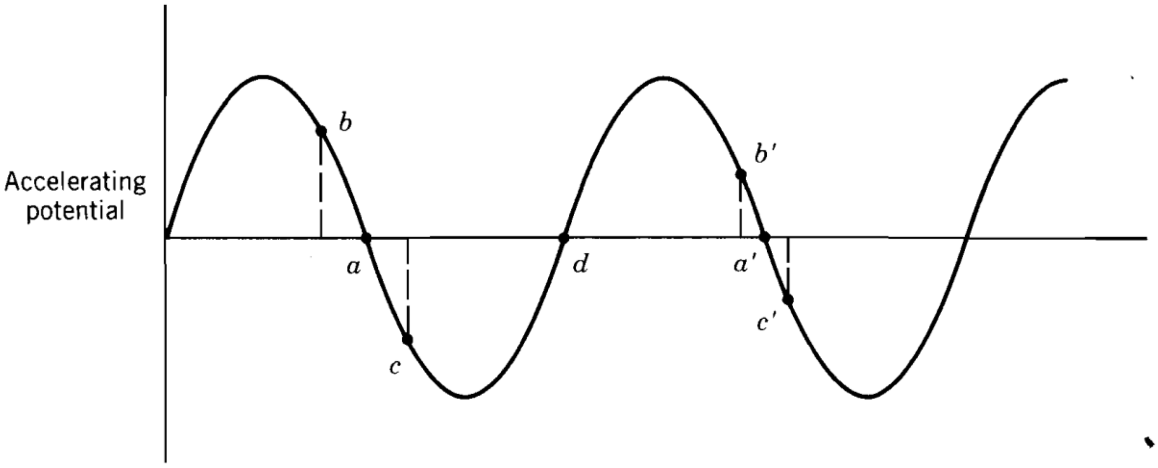

Fig. 3

Now extending the cyclotron or synchrocyclotron to higher energy means building machines of larger radii. Because the magnet is the principal factor in the cost of a cyclotron, we expect that costs of building larger cyclotrons will be very high.

The solution to this dilemma is the synchrotron accelerator, in which both the magnetic field strength and the resonant frequency are varied. Fig. 4 shows the simplest design for a synchrotron. The essential feature that keeps the costs reasonable as the energy is extended is that particles orbit at a very nearly constant radius at high energies. The magnetic field therefore need be applied only at the circumference, not throughout the entire circular volume, as in an ordinary cyclotron. An annular magnet, as shown in Fig. 4, accomplishes the task. Particles follow a circular path and are accelerated by a resonant electric field as they cross a gap during each orbit. As the energy increases, the frequency of the AC voltage across the gap must increase to maintain the resonance. Simultaneously, the magnetic field must increase to keep the radius constant.

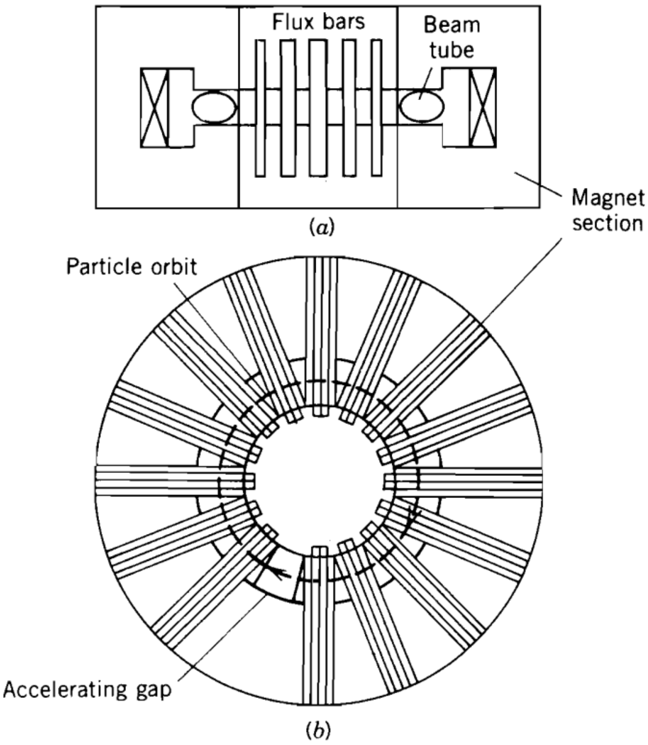

Fig. 4

In a magnetic field of strength B, a particle of charge q moves in a circular arc of radius r given by the cyclotron expression $r = \frac{mv}{qB} = \frac{p}{qB}$, where p is the momentum of the particle. Therefore, $p = qrB$. The total relativistic energy of the particle is given by

$$E = \sqrt{p^2c^2 + {m_0}^2c^4} = \sqrt{q^2r^2B^2c^2 + {m_0}^2c^4}$$

The basic cyclotron resonance condition, can then be written for the frequency of the AC voltage (f) as

$$f = \frac{qB}{2\pi m} = \frac{qBc^2}{2\pi E}$$

since $E = mc^2 = \sqrt{q^2r^2B^2c^2 + {m_0}^2c^4}$. We therefore can write

$$f = \frac{qBc^2}{2\pi\sqrt{q^2r^2B^2c^2 + {m_0}^2c^4}}$$

For a given r the previous equation gives the relationship between B and f necessary to maintain the synchronization.

Linear Accelerators (or Linacs):

In a linear accelerator (also known as a linac), particles receive many individual accelerations by an AC voltage, exactly like in a cyclotron. The difference here is that they travel in a straight line in a linac. This immediately eliminates the large costs of cyclotron magnets and the defocusing effects associated with magnetic fields.

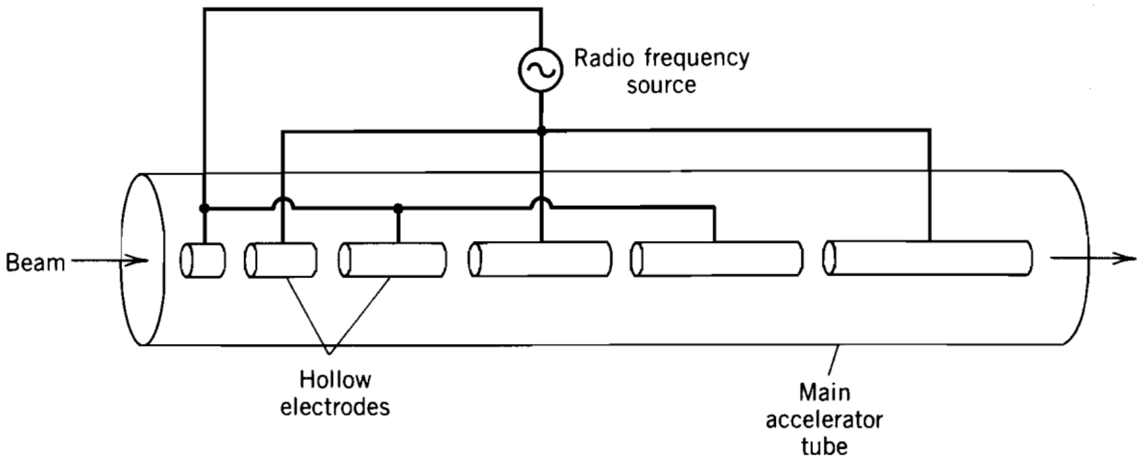

Fig. 5

The basic design of a linac is illustrated in Fig. 5. The beam travels through a series of hollow cylindrical electrodes connected *alternately* to *opposite poles* of the AC voltage source. Particles are accelerated as they cross the gap between the electrodes. Upon entering the interior of an electrode, the particles drift in a field-free region (hence the name *drift tube* given to the electrodes) for a time equal to half the period of the AC voltage. In this way the polarity of the voltage is reversed during the time the particles are within the drift tube, and it is then accelerated as it crosses the next gap.

The operation of such an accelerator is dependent on the condition that the entrance of the particles into each gap be in resonance with the electric field across the gap. If T is the time period of the AC voltage, then the length of the nth drift tube for particles of velocity v_n should be given by $L_n = v_n T/2$.

For non-relativistic particles of charge q with zero initial velocity, after passing through n gaps of voltage difference V_0, the kinetic energy can be written as

$$\frac{1}{2} m {v_n}^2 = nqV_0$$

Therefore, we obtain $v_n = \sqrt{\frac{2nqV_0}{m}}$. So, we can write $L_n = \sqrt{\frac{nqV_0}{2m}} T \propto \sqrt{n}$.

The drift-tube lengths must therefore increase as $\sqrt{n}$. For relativistic particles, where $v \cong c$, the drift-tube lengths are roughly constant.

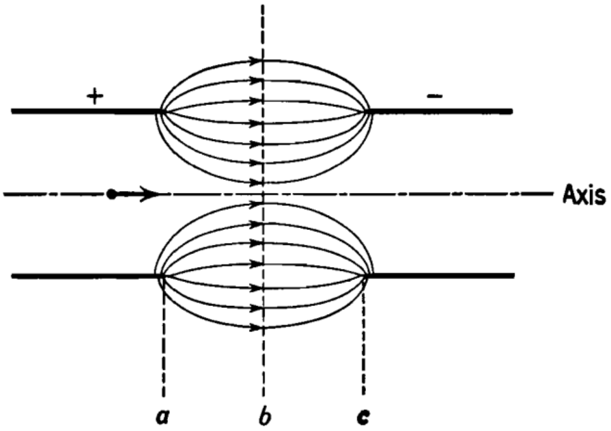

Fig. 6

Upon crossing the gap, particles will experience a slight radial focusing, which can be understood with reference to Fig. 6. In the left half of the gap (region *ab*), the lines of force of the electric field focus off-axis particles toward the axis, while in region *bc*, there is a defocusing effect. However, the acceleration of the particles means they move more slowly and thus spend more time, in region *ab* so that the focusing effect exceeds slightly the defocusing effect. This slight focusing (which would occur for static fields) is altered by the time-varying nature of the field, the effect of which can be discussed with reference to the consideration of phase stability.

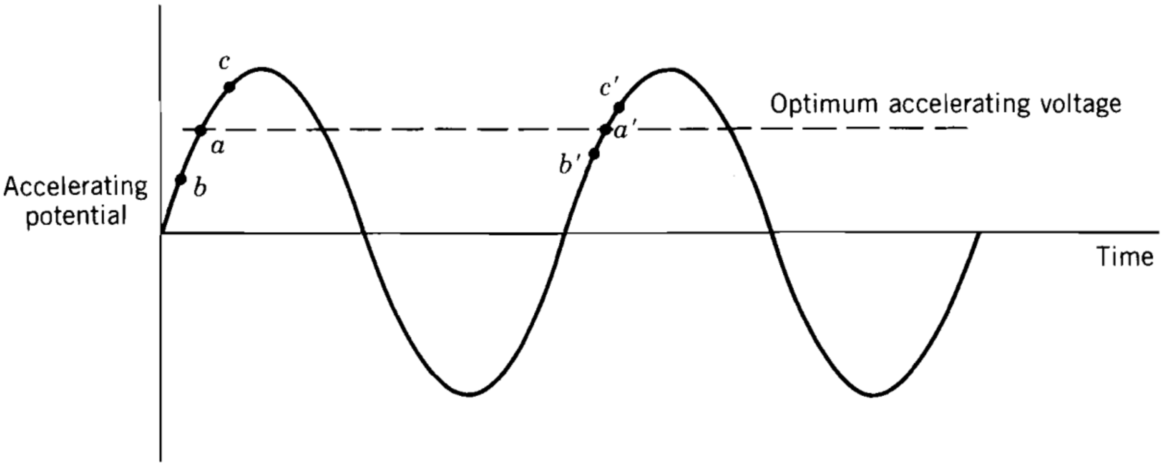

Fig. 7

Phase stability in a linac is achieved when the bunch arrives on the increasing phase of the AC cycle as shown in Fig. 7. Let us consider a bunch of particles arriving at the gap. Because the voltage is rising, particles arriving early (at the front of the bunch) do not experience the optimum voltage. They are accelerated somewhat less than the particles arriving later and they take longer to cross the drift tube. These “early” particles are thus delayed and arrive toward the centre or even the end of the bunch at the next gap. Similarly, particles that reach the gap near the end of a bunch experience a larger voltage and a greater acceleration, which pushes them toward the beginning of the next bunch. For each bunch there is an optimum voltage for perfect resonance, about which the particles may oscillate from one gap to the next, but the net result is a phase stability that keeps the bunches together.

Dr. Avradip Pradhan,
Assistant Professor,
Department of Physics,
Narajole Raj College, Narajole.

Reference(s):

Concepts of Nuclear Physics, Bernard L. Cohen, McGraw-Hill

Introductory Nuclear Physics, Kenneth S. Krane, John Wiley & Sons

(All the figures have been collected from the above mentioned references)

PAPER: DSE2T (Nuclear and Particle Physics)
TOPIC(s): Particle Accelerators (Part – 2)