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C6T (Thermal Physics) , Topic :- Kinetic Theory of Gases(Part-1)

❖ Introduction :

In 1827 Robert Brown observed a kind of motion exhibited by Pollen grains when suspended in water. This is called Brownian motion. Under a powerful microscope, he noticed that the pollen grains in the colloidal solution show a rapid and continuous to and –fro motion- a kind of dancing in the wildest fashion. Each grain spins, rises,sinks and rises again. The motion never ceases- it is eternal and spontaneous.

❖ Assumptions of the Kinetic Theory:

To develop the kinetic theory of gases, the following idealized assumptions have been made.

(i) A gas consists of a large number of identical molecules which are like minute hard elastic spheres constantly moving about in all possible directions with different velocities ranging from zero to infinity in a random motion.

(ii) During random motion, the molecules collide with one another and also with the walls of the container elastically,i.e., no loss of kinetic energy takes place during the collisions. As the chance of collision in all directions is the same, the collisions do not affect the molecular density.

(iii) The molecule do not exert any force (attraction and repulsion) on each other except when they actually collide.

(iv) Between two successive collisions they move in straight line with uniform velocity .The distance traversed between two successive collisions is known as the free path. The average of all the free paths is called the mean free path.

(v) The collisions are essentially instantaneous .i.e, the duration of a collision is insignificant in comparison to the time between the collisions.

(vi) Since the molecule are like geometrical mass-points, the actual volume occupied by the molecule is negligible compared to the total volume of the gas.

- **Deduction of Perfect Gas Equation:**

Let us consider a perfect gas enclosed in a container. One of the surface of the container is perpendicular to the X-axis. In the following using figure, BAE represents the surface.

Now, any velocity c of a particle in space can be resolved into components u, v and w along three axes such that,

$$c^2 = u^2 + v^2 + w^2 \qquad \ldots(1)$$

The limits of u, v and w are from $-\infty$ to $+\infty$ and those of c are from 0 to ∞.

Now, only u suffers change in direction on reflection from the wall BAE, the other two components v and w remain unchanged by this type of reflection.

.: The change in momentum per reflection of a molecule is

$$= mu — mu = 2mu$$

Suppose, n_u is the number of molecules per unit volume moving with velocity u. So, the number of molecules striking unit area of the wall in time dt would be contained in a cylinder of area unity and vertical height udt.

.: Volume of the cylinder = udt

.: Number of molecules in the cylinder = $n_u udt$

.: Change in momentum per unit area suffered by the above molecules in time dt is

$$= 2mu \times n_u udt = 2mn_u u^2 dt$$

.: The total change in momentum per unit area is

$$= 2m \sum_{u=0}^{\infty} n_u u^2 dt$$

[If the above change in momentum results in an average force $\underset{\partial F}{\rightarrow}$ then we can write]

$$\underset{\partial F}{\rightarrow}.dt = 2m \sum_{u=0}^{\infty} n_u u^2 dt \qquad \ldots..(2)$$

Since the involed is unity, $\underset{\partial F}{\rightarrow}$ will represent the pressure (p)

$$p = 2m \sum_{u=0}^{\infty} n_u u^2 dt \qquad \ldots..(3)$$

Where n_u is a function of u.
Let $\overline{u^2}$ be the mean square velocity. So, we may write

$$\overline{u^2} = \frac{n_n u_1^2 + n_2 u_1^2 + \cdots + n_i u_i^2 + \cdots}{n_1 + n_2 + \cdots + n_i + \cdots} = \frac{\sum_i n_i u_i^2}{\sum_i n_i} = \frac{\sum_i n_i u_i^2}{n/2} \qquad \ldots(4)$$

The factor $\frac{1}{2}$ arises due to the fact that only the molecule in the positive x-direction are being considered.

$$\sum_{u=0}^{\infty} n_u u^2 = \frac{1}{2} n\overline{u^2}$$; n being total number of molecules per cc

Similarly

$$p_x = 2m \times \frac{1}{2} n\overline{u^2} = mn\overline{u^2}$$

$$p_y = 2m \times \frac{1}{2} n\overline{v^2} = mn\overline{v^2}$$

$$p_z = 2m \times \frac{1}{2} n\overline{w^2} = mn\overline{w^2}$$

.: The expression for the pressure is

$$p = p_x = p_y = p_z = mn\overline{u^2} = mn\overline{v^2} = mn\overline{w^2}$$

But $\overline{u^2} = \overline{v^2} = \overline{w^2} = \frac{1}{3}\overline{c^2}$; where $\overline{c^2}$ is the mean square velocity of the particles.

$$.\!:\! p = \frac{1}{3} mn\overline{c^2} \qquad \ldots..(6)$$

Where , C=$\sqrt{\overline{c^2}}$ = root mean square velocity (r.m.s velocity) of the molecules.

The above expression may be also be written as,

$$\boldsymbol{p = \frac{1}{3}\rho c^2}$$

Where $\rho = mn =$ density of the gas.

❖ **Concept of Probability :** The probability of an event is the limit to which the ratio of the number of trails resulting in occurrence of the event to the total number of trials upon infinite growth of the latter tends.

Let N' be the number of occurrence of the event we are interested in and N be the total number of trials , then the probability of occurrence of the event is

$$P = \lim_{N\to\infty} \frac{N'}{N}$$

❖ **Summation of Probabilities :**The theorem of summation of probabilities states that if $P_1, P_2, P_3, \ldots\ldots$ are the probabilities of several mutually exclusive events , then the probabilities that one of them will happen is the sum of the probabilities of happening for the separate events,

i.e $P = P_1 + P_2 + P_3 + \cdots$

Example: Suppose a box contains 10 balls which are all identical, but 3 white and the rest black. So, the probability of extracting any ball (white or black) is $\frac{1}{10}$, Since there are 10 balls. Now, the number of white balls is 3 and which one of them is being extracted is immaterial. So, the probabilities of extracting a white is

$$P = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}$$

❖ **Multiplication of Probabilities** : The probabilities of the simultaneous happening of two or more independent events equals the product of the probabilities of each of them happening separately, i.e,

$$P_{123} = P_1 \times P_2 \times P_3$$

Where $P_{123}\ldots$ is the probabilities of the composite event and $P_1P_2P_3\ldots..$ are the probabilities of independent events happening separately.

Example: Suppose, there are five boys A,B,C,D and E. If there is black ball, the probability that A gets it is $\frac{1}{5}$, Since any one of the five boys is equally likely to get it. Similarly if there is a white ball, the chance of a getting it is also $\frac{1}{5}$. But , the probability that A may get both the balls simultaneously is,

$$= \frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$$

Because there are now in all 25 equally likely ways in which the composite event can take place and the desired event takes place only once.