C13T (Electromagnetic Theory)

Topic – Polarization of Electromagnetic Waves (Part – 3)

We have already discussed part 2 of this e-report.

Now let us continue part 3 of it.

Production and Detection of Plane Polarized Light, Circularly Polarized Light and Elliptically Polarized Light:

Production. When a monochromatic unpolarized light passes through a Nicol Prism, then it splits into an o-wave and e-wave. These waves are plane polarized. Inside the crystal, when these rays are incident on a Canada Balsam layer. It allows only the e-wave to emerge from it. In this way, the light emerging from the Nicol prism is linearly or plane polarized with vibrations parallel to the principal section.

When the plane polarized light emergent from a Nicol Prism is allowed to fall on a quarter-wave plate such that its vibrations make an angle of θ with the optic axis of quarter-wave plate, then

1. For \( \theta = 0^0 \) and \( 90^0 \), the emergent light remains linearly or plane polarized,
2. For \( \theta = 45^0 \), the emergent light becomes circularly polarized and
3. For The values of \( \theta \) other than \( 0^0, 45^0 \) and \( 90^0 \) the emergent light becomes elliptically polarized.

Detection. Using a polarizer and a quarter-wave plate, the state of polarization can be determined. The following steps are used in the analysis for the detection of the state of polarization of a given light beam.

1. A rotating polarizer is placed into the path of the given light as shown in Fig. 1. If the intensity of the transmitted light varies with zero minima twice in one complete rotation, then the incident light will be plane polarized.

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2. If the intensity of the transmitted light varies with non-zero minima twice in one complete rotation as shown in Fig. 2, then the incident light will be either elliptically polarized or partially polarized.

3. If the intensity of the transmitted light remains constant in one complete rotation as shown in Fig. 3, then the incident light will be either circularly polarized or unpolarized.

To distinguish between the elliptically polarized and partially polarized light or between the circularly polarized or unpolarized light, the incident light is first focused on a quarter-wave plate and then allowed to pass through the polarizer.

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If the incident light is elliptically polarized, the quarter-wave plate converts it into a plane polarized light. Hence the intensity of the transmitted light varies with zero minima twice in one complete rotation. If the incident light is partially polarized, then the intensity of the transmitted light varies with non-zero minima twice in one complete rotation.

Optical Activity, Rotatory Polarization:

The manner in which light interacts with material substances can yield a great deal of valuable information about their molecular structures. The process to be examined next, although of specific interest in the study of Optics, has had and is continuing to have far-reaching effects in the sciences of Chemistry and Biology. In 1811 the French physicist Dominique F. J. Arago first observed the rather fascinating phenomenon now known as optical activity. It was then that he discovered that the plane of vibration of a beam of linear light underwent a continuous rotation as it propagated along the optic axis of a quartz plate (Fig. 4).

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Dextrorotatory and levorotatory. At about the same time Jean Baptiste Biot saw this same effect while using both the vaporous and liquid forms of various natural substances like turpentine. Any material that causes the electric field of an incident linear plane wave to appear to rotate is said to be optically active. Moreover, as Biot found, one must distinguish between right and left-handed rotation. If while looking in the direction of the source, the plane-of-vibration appears to have revolved clockwise, the substance is referred to as dextrorotatory, or $d$ -rotatory (from the Latin dextro, meaning right) or right-handed. Alternatively, if the electric field appears to have been displaced counter clockwise, the material is levorotatory, or $l$ -rotatory (from the Latin levo, meaning left) or left-handed.

Sir John F. W. Herschel recognized that $d$ -rotatory and $l$ -rotatory behaviour in quartz actually corresponded to two different crystallographic structures. Although the molecules are identical (SiO$_2$), crystal quartz can be either right- or left-handed, depending on the arrangement of those molecules. The external appearances of these two forms are the same in all respects, except that one is the mirror image of the other; they are said to be enantiomorphs of each other.
All transparent enantiomorphic substances are optically active. Furthermore, molten quartz and fused quartz, neither of which is crystalline, are not optically active. Evidently, in quartz optical activity is associated with the structural distribution of the molecules as a whole. There are many substances, both organic and inorganic (e.g. benzil and NaBrO₃, respectively), which, like quartz, exhibit optical activity only in crystal form. In contrast, many naturally occurring organic compounds, such as sugar, tartaric acid, and turpentine, are optically active in solution or in the liquid state. Here the rotatory power, as it is often referred to, is evidently an attribute of the individual molecules. There are also more complicated substances for which optical activity is associated with both the molecules themselves and their arrangement within the various crystals. An example is rubidium tartrate. A d —rotatory solution of that compound will change to l —rotatory when crystallized.

### Biot’s Laws of Optical Rotation:

In 1815, Biot studied the optical rotation and proposed the following laws for it.

1. For the light of a particular wavelength, the optical rotation \( \theta \) of its plane of polarization by an optically active substance is directly proportional to the length \( l \) of the substance through which the polarized light passes, i.e. \( \theta \propto l \).

2. The optical rotation \( \theta \) produced by different optically active substances is equal to the algebraic sum of the optical rotations produced by the individual substances. The clockwise and anticlockwise rotations are taken of opposite signs, i.e. \( \theta = \theta_1 + \theta_2 + \theta_3 + \ldots \) where \( \theta_1, \theta_2, \theta_3 \) etc. are taken with proper signs.

3. The optical rotation \( \theta \) produced by a solution or vapour of an optically active substance is directly proportional to its concentration \( c \), i.e. \( \theta \propto c \).

4. The optical rotation \( \theta \) produced by a particular thickness of an optically active substance is inversely proportional to the wavelength of the experimental light, i.e. \( \theta \propto \lambda^{-1} \).
Fresnel’s Theory of Optical Rotation:

Fresnel, without addressing the actual mechanism involved, proposed a simple phenomenological description of optical activity. Since the incident linear wave can be represented as a superposition of $R-$ and $L-$states, he suggested that these two forms of circular light propagate at different speeds. An active material shows circular birefringence; that is, it possesses two indices of refraction, one for $R-$states ($n_R$) and one for $L-$states ($n_L$). In traversing an optically active specimen, the two circular waves would get out-of-phase, and the resultant linear wave would appear to have rotated. We can see how this is possible analytically by describing monochromatic right- and left-circular light propagating in the $z-$direction. It was seen that the sum of these two waves is indeed linearly polarized. We now alter these expressions slightly in order to remove the factor of two in the amplitude in which case

$$\vec{E}_R = \frac{E_0}{2} [i \cos(k_R z - \omega t) + j \sin(k_R z - \omega t)]$$

and

$$\vec{E}_L = \frac{E_0}{2} [i \cos(k_L z - \omega t) - j \sin(k_L z - \omega t)]$$

represent the right- and left-handed constituent waves. Since $\omega$ is constant, $k_R = k_0 n_R$ and $k_L = k_0 n_L$. The resultant disturbance is given by $\vec{E} = \vec{E}_R + \vec{E}_L$, and after a bit of trigonometric manipulation, it becomes

$$\vec{E} = E_0 \cos[(k_R + k_L) \frac{z}{2} - \omega t] [i \cos(k_R - k_L) \frac{z}{2} + j \sin(k_R - k_L) \frac{z}{2}]$$

At the position where the wave enters the medium ($z = 0$) it is linearly polarized along the $x-$axis, so $\vec{E} = E_0 i \cos \omega t$.

It is important to notice that at any point along the path, the two components have the same time dependence and are therefore in phase. This just means that anywhere along the $z-$axis the resultant is linearly polarized (Fig. 5), although its orientation is certainly a function of $z$. Moreover, if $n_R > n_L$ or, equivalently, $k_R > k_L$, $\vec{E}$ will rotate counter clockwise, whereas if $n_L > n_R$, the rotation is clockwise (looking toward the source). Traditionally, the angle $\theta$
through which $\mathbf{E}$ rotates is defined as positive when it is clockwise. Keeping this sign convention in mind, it should be clear that the field at point $z$ makes an angle of $\theta = -(k_R - k_L) \frac{z}{2}$ with respect to its original orientation. If the medium has a thickness $d$, the angle through which the plane-of-vibration rotates is then given by

$$\theta = \frac{\pi d}{\lambda_0} (n_L - n_R)$$

where $n_L > n_R$ is $d$-rotatory and $n_R > n_L$ is $l$-rotatory. This is how the optical rotation angle is calculated.

**Fig. 5**

**Specific Rotation.** The specific rotation ($S$) of an optically active substance at a given temperature and for a given wavelength of light, is defined as the rotation (in degrees) produced by 1 decimetre length of the substance in solution when its concentration is 1 gcm$^{-3}$. Mathematically, it can be expressed as

$$S = \frac{\theta}{lc}$$

where $\theta$ is the optical rotation in degrees, $l$ is the length of the solution in decimetres, $c$ is the concentration of the solution.

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Laurent’s Half-shade Polarimeter:

It is an optical device, which is used for measuring the optical rotation of optically active substances. Given below is the brief discussion for the construction and working of the Laurent's half-shade polarimeter.

**Structure.** The constituent parts of the device are described below (Fig. 6).

- O is a monochromatic light source,
- S is a slit to obtain the monochromatic light emitted from O in a particular direction,
- L is a convex lens to convert the monochromatic light emitted from O in the form of a parallel beam,
- \( N_1 \) is a Nicol Prism which acts as a polarizer,
- AB is a circular plate, known as the Laurent’s plate. Its left-half part is made of glass and right-half part is made of quartz,
- G is a hollow tube for filling the optically active substance,
- \( N_2 \) is a Nicol Prism which acts as an analyzer and
- T is a telescope to observe the optical rotation.

![Diagram of the Laurent’s Half-shade Polarimeter](image)

**Working Principle.** The light passing through the polarizer \( N_1 \) is incident normally on Laurent's plate AB with its vibration along OP as shown in Fig. 7. On passing through its semicircle glass-half part ACB, its vibrations remain along OP, but on passing through the semicircle quartz-half part ADB, the light

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ray is split into o-ray and e-ray. The vibrations of o-ray and e-ray are along OD and OA respectively. The semicircle quartz-half part acts as $\frac{\lambda}{2}$ plate and introduces a phase difference of $\pi$ between their vibrations. Hence on emergence from the plate AB, vibrations of o-ray occur along OC instead of OD. The resultant vibration on emerging will, therefore, be along OP′ such that $\angle POA = \angle P′OA$.

Fig. 7

Now when we analyze the light coming out of the analyzer N2, the following observations are relevant.

1. If the principal plane of N2 is parallel to OP, the glass-half part will appear brighter than the quartz-half part.

2. If the principal plane of N2 is parallel to OP′, the quartz-half part will appear brighter than the glass-half part.

3. If the principal plane of N2 is parallel to AOB, the quartz-half part and the glass-half part will appear equally bright.

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4. If the principal plane of $N_2$ is normal to AOB, the quartz-half part and the glass-half part will appear equally less bright compared to the observation 3.

Reference(s):

Applied Physics for Engineers, Neeraj Mehta, Prentice Hall India

Optics, Eugene Hecht, Pearson Education

(All the figures have been collected from the above mentioned references)