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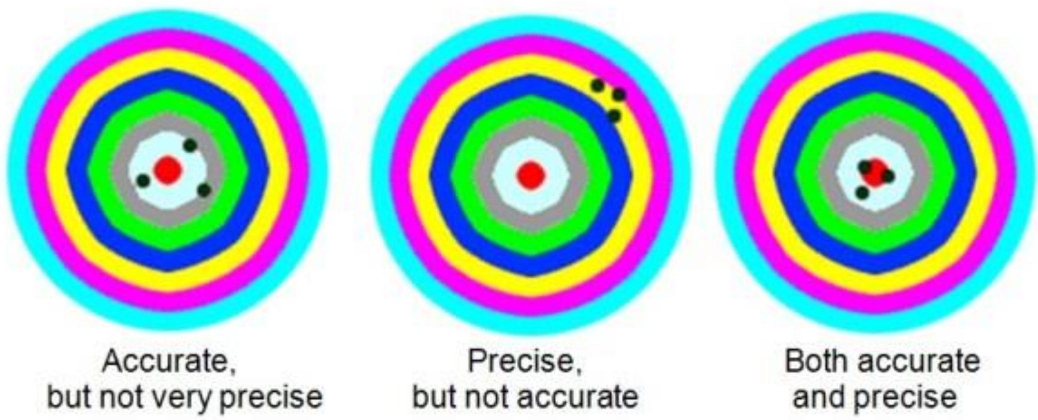
Topic:

Measurement: Accuracy and precision. Significant figures. Error and uncertainty analysis. Types of errors: Gross error, Systematic error, Random error. Statistical analysis of data (Arithmetic mean, deviation from mean, average deviation, standard deviation, chi-square) and curve fitting. Guassian distribution.

Experimental Techniques: Measurement

Accuracy and Precision

Accuracy is how close a measured value is to the**actual (true) value**.Precision is how close the measured values areto each other.



Significant Figure

The significant figures of a number are digits that carry meaningful contributions to its measurement resolution. This includes all digits except.

Property

i. All non-zero numbers ARE significant. The number 59.6 has THREE significant figures because all of the digits present are non-zero.
ii. Zeros between two non-zero digits are significant.6072 has FOUR significant figures. The zero is between a 6 and a 7.
iii. Leading zeros are not significant. The number 0.89 has only two significant figures. 0.0086 also has two significant figures. All of the zeros are leading.
iv. Trailing zeros to the right of the decimal are significant. There are four significant figures in 92.00.

v. Trailing zeros in a whole number with the decimal shown are significant "670." indicates that the trailing zero is significant; there are three significant figures in this value.

vi. Trailing zeros in a whole number with no decimal shown are not significant. Writing just "670" indicates that the zero is NOT significant, and there are only TWO significant figures in this value.

vii. Exact numbers have an infinite number of significant figures. This rule applies to numbers that are definitions. For example, 1 meter = 1.00 meters = 1.0000 meters =1.0000000000000000000 meters, etc.

viii. For a number in scientific notation: N x 10^x, all digits comprising N are significant by the first 6 rules; "10" and "x" are not significant.5.02 x 10^4has three significant figures: "5.02." "10 and "4" are not significant.

Error Analysis

All measurements have some degree of uncertainty. The process of assessing the uncertainty associated with a measurement result is called uncertainty analysisorerror analysis.

$$\text{Measurement} = (\text{Best Estimate} \pm \text{Uncertainty})$$

As for example, if the digital display of the balance is limited to 2 decimal places a weight of 18.42 becomes m= 18.42 ± 0.01 g.

Accuracy is the closeness of agreement between a measured value and a true or accepted value. Measurement Error is the amount of inaccuracy.

Precisionis a measure of how well a result can be determined. It is the degree of consistency and agreement among independent measurements of the same quantity. This is the reproducibility of the measurements.

The **uncertainty** should account for both the accuracy and precision of the measurement.

$$\text{Relative Uncertainty} = \left|\frac{\text{Uncertainty}}{\text{Measured Quantity}}\right|$$

Example:m= 95.5 ± 0.5 g

$$\text{Relative Uncertainty} = \left|\frac{\text{Uncertainty}}{\text{Measured Quantity}}\right| = \left|\frac{0.5\ gm}{95.5}\right| = 0.005 = 0.5\%$$

$$\text{Relative Error} = \frac{\text{Measured Value} - \text{Expected Value}}{\text{Expected Value}}$$

If the expected value formis 100.0 g, then the relative error is:

Relative Error $=\frac{95.5-100.0}{100} = -0.045 = -4.5\%$

Gross Error

Gross errors are caused by experimenter carelessness or equipment failure. These "outliers" are so far above or below the true value that they are usually discarded when assessing data. The "Q-Test" is a systematic way to determine if a data point should be discarded.

The quantity Q is the absolute difference between the questioned measurement (x_q) and the next closest measurement (x_n) divided by the spread (ω), the difference between the largest and smallest measurement, of the entire set of data.

$$Q = \frac{(x_q - x_n)}{\omega}$$

Q is compared to a specified confidence levels (the percent probability a measurement will fall into a range around the mean (x). If Q is greater than the values for a particular confidence level, the measurement should be rejected. If Q is less than the values in the table, the measurement should be retained.

Types of Error

There are two types of error-i) Random Error and ii) Systematic Error

Random Errors- Statistical fluctuations (in either direction) in the measured data due to the precision limitations of the measurement device. Random errors can be evaluated through statistical analysis and can be reduced by averaging over a large number of observations.

The random error (also called the mean deviation) is then a measure of the spread of the repeat readings: Random error, $\Delta_{Ran} = \frac{R}{N}$= R = range (maximum - minimum) N = number of repeat readings Random error is reduced by increasing the number of readings, N. As N increases Δ_{Ran}decreases.

Systematic Errors - Reproducible inaccuracies that are consistently in the same direction. These errors are difficult to detect and cannot be analysed statistically.

Example: To find the acceleration due to gravity using an object that is subject to significant air friction. It will have the effect of shifting all results by a significant amount in the same direction, known as the systematic error

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Average Mean

$$\text{Average (Mean)} = \frac{x_1+x_2+\cdots\ldots\ldots x_n}{N}$$

Example:

Observation	Length
1	33.26
2	33.20
3	33.25
4	33.22
5	33.21

Average= $\frac{33.26+33.20+33.25+33.22+33.21}{5}$ =33.23

$$\text{Average Deviation } (\bar{d}) = \frac{|x_1-\bar{x}|+|x_2-\bar{x}|+\cdots\ldots\ldots+|x_n-\bar{x}|}{N}$$

Standard Deviation:

Procedureto calculate the standard deviation for a sample of N measurements:

i. Sum all the measurements and divide by N to get the average, ormean.
ii. Subtract this average from each of the N measurements to obtain "deviations".
iii. Square each of these N deviations and add them all up.
iv. Divide this result by (N− 1) and take the square root.

We can write out the formula for the standard deviation as follows. Let the N measurements be called, $x_1, x_2, \ldots\ldots\ldots\ldots x_n$. Let the average of the N values be called $\bar{x}$.Then each deviation is given by $\delta x_i = x_i - \bar{x_i}$ for $i = 1,2,3 \ldots\ldots n$

$$\text{Standard Deviation } (S) = \sqrt{\frac{(\delta x_1^2+\delta x_2^2+\cdots\ldots+\delta x_n^2)}{(N-1)}} = \sqrt{\frac{\sum \delta x_i^2}{(N-1)}}$$

In previous example $\bar{x}$ = 33.23

Observation	Length	Deviation (cm)
1	33.26	+0.03=33.26-33.23
2	33.20	-0.03=33.20-33.23
3	33.25	+0.02=33.25-33.23
4	33.22	-0.01=33.22-33.23
5	33.21	-0.02=33.21-33.23

The standard deviation (s) = $\sqrt{\frac{(0.03)^2+(0.03)^2+(0.02)^2+(0.01)^2+(0.02)^2}{(5-1)}} = 0.0258\ cm$

Gaussian distribution:

Normal distribution is a continuous probability distribution. Any quantity whose variation depends on random causes will be distributed according to normal distribution.

Normal distribution can be derived from binomial distribution in the limiting case when the number of trial n is very large and the probability of success is close to $\frac{1}{2}$. The normal (or, Gaussian) distribution is defined as

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x-\mu)}{2\alpha^2}} , -\infty < x < \propto$$

Where the mean μ and the standard deviation $\sigma > 0$ are arbitrary constants. $x$is the normal variate and $f(x)$ is the probability density function of the normal distribution.

Here,

$$\int_{-\infty}^{+\infty} f(x)dx = 1$$

$f(x)\ is\ called\ probablity\ function, if$

$f(x) = 0\ for\ every\ value\ of\ x$

$$\int_{x_1}^{x_2} f(x)dx = P\ (x_1 < x < x_2)$$

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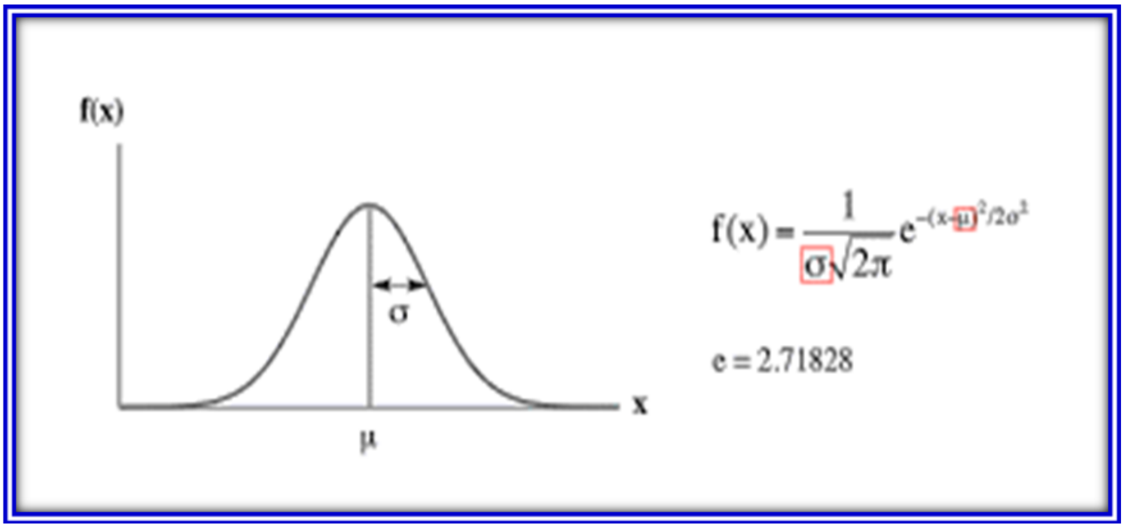

Chi Square:

For fitting a hypothesized function to a set of experimental data points. Such procedures involve minimizing a quantity we called Φ in order to determine best estimates for certain function parameters, such as (for a straight line) a slope and an intercept. Φ is proportional to (or in some cases equal to) a statistical measure called χ^2, or chi-square, a quantity commonly used to test whether any given data are well described by some hypothesized function.

Such a determination is called a chi-square test for goodness of fit. In the following, we discuss χ^2 and its statistical distribution, and show how it can be used as a test for goodness of fit.1

Definition of Chi Square (χ^2):

$$\chi^2 = \frac{(x_1-\mu_1)^2}{\sigma_1^2}+\frac{(x_2-\mu_2)^2}{\sigma_2^2}+.........+\frac{(x_\vartheta-\mu_\vartheta)^2}{\sigma_\vartheta^2}\sum_{i=1}^{\vartheta}\frac{(x_i-\mu_i)^2}{\sigma_i^2}$$

Here, the random fluctuations of the values of x_i about their mean values μ_i , each term in the sum will be of order unity. If,μ_i and the σ_i correctly, a calculated value of χ^2 will be approximately equal to v.

Properties

The following conditions are maintained-

a set of measurements $\{x_1, x_2, \ldots x_n\}$.

the true value of each x_i $(x_{t1}, x_{t2}, \ldots x_{tn})$

the closer the $(x_1, x_2, \ldots x_n)$'s are to the $(x_{t1}, x_{t2}, \ldots x_{tn})$'s.

better (or more accurate) the measurements

measurements are independent of each other.
measurements come from a Gaussian distribution

$(\sigma_1, \sigma_2 \dots \sigma_n)$ be the standard deviation associated with each measurement

Consider the following two possible measures of the quality of the data:

$$R \equiv \sum_{i=1}^{n} \frac{x_i - x_{ti}}{\sigma_i}$$

$$\chi^2 \equiv \sum_{i=1}^{n} \frac{(x_i - x_{ti})^2}{\sigma_i^2}$$

Chi Square (χ^2) *distribution*:

One can show that the probability distribution for χ^2 is exactly:

$$p(\chi^2, n) = \frac{1}{2^{n/2}\Gamma(n/2)}[\chi^2]^{n/2-1} e^{-\chi^2/2} \qquad 0 \le \chi^2 \le \infty$$

- This is called the "Chi Square" (χ^2) distribution.

Γ is the Gamma Function:

$$\Gamma(x) \equiv \int_0^\infty e^{-t} t^{x-1} dt \qquad x > 0$$

$$\Gamma(n+1) = n! \qquad n = 1, 2, 3\dots$$

$$\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$$

Curve Fitting:

Curve fitting is the process of constructing a curve, or mathematical function that has the best fit to a series of data points, possibly subject to constraints. The curve fitting can involve either interpolation, where an exact fit to the data is required, or smoothing in which a "smooth" function is constructed that approximately fits the data.

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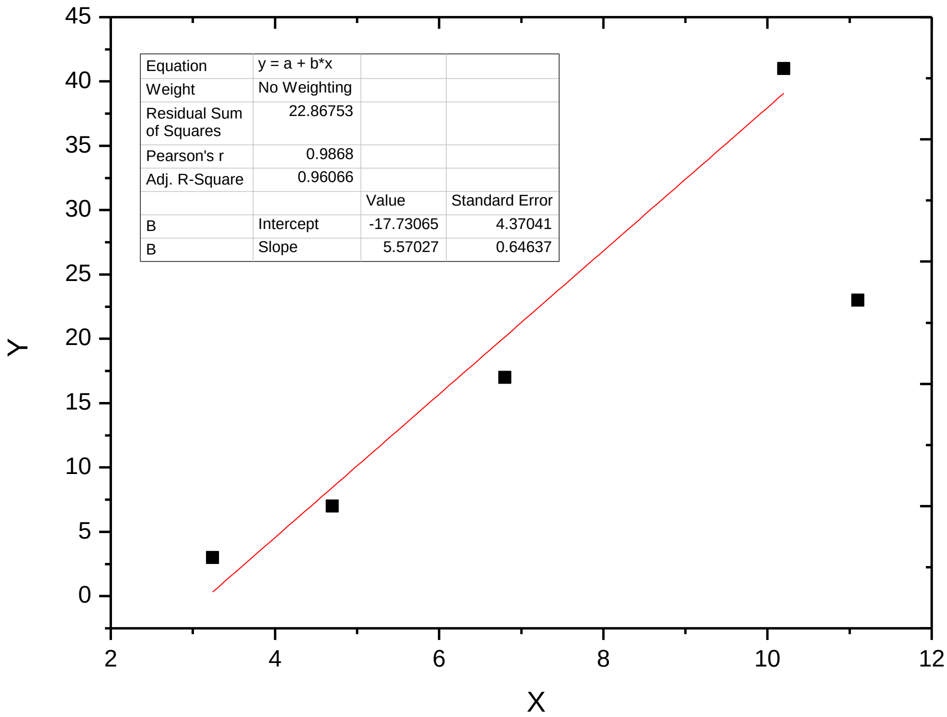

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(ix) Taylor, John. An Introduction to Error Analysis, 2nd. ed. University Science Books: Sausalito, 1997.
(x) https://www.rdatagen.net/post/a-little-intuition-and-simulation-behind-the-chi-square-test-of-independence-part-2/
(xi) Lecture, K. K. Gan

Link to Audio visual Lectures (e-Lectures) on this topic given by Distinguish Professors of Indian & Foreign Universities:

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(i)https://www.khanacademy.org/math/statistics-probability/inference-categorical-data-chi-square-tests/chi-square-goodness-of-fit-tests/v/pearson-s-chi-square-test-goodness-of-fit

(ii) https://www.youtube.com/watch?v=3aRIwDDMc88

(iii) https://www.youtube.com/watch?v=X0Tuq1qQdKQ

(iv)https://nptel.ac.in/content/storage2/courses/downloads/102101056/Assignment-3_noc18_bt21_83.pdf

(v) https://nptel.ac.in/content/storage2/nptel_data3/html/mhrd/ict/text/111104120/lec20.pdf

(vi) https://www.youtube.com/watch?v=8UY7ow62Wqs

(vii)https://www.youtube.com/watch?v=fvgDqVda9L8