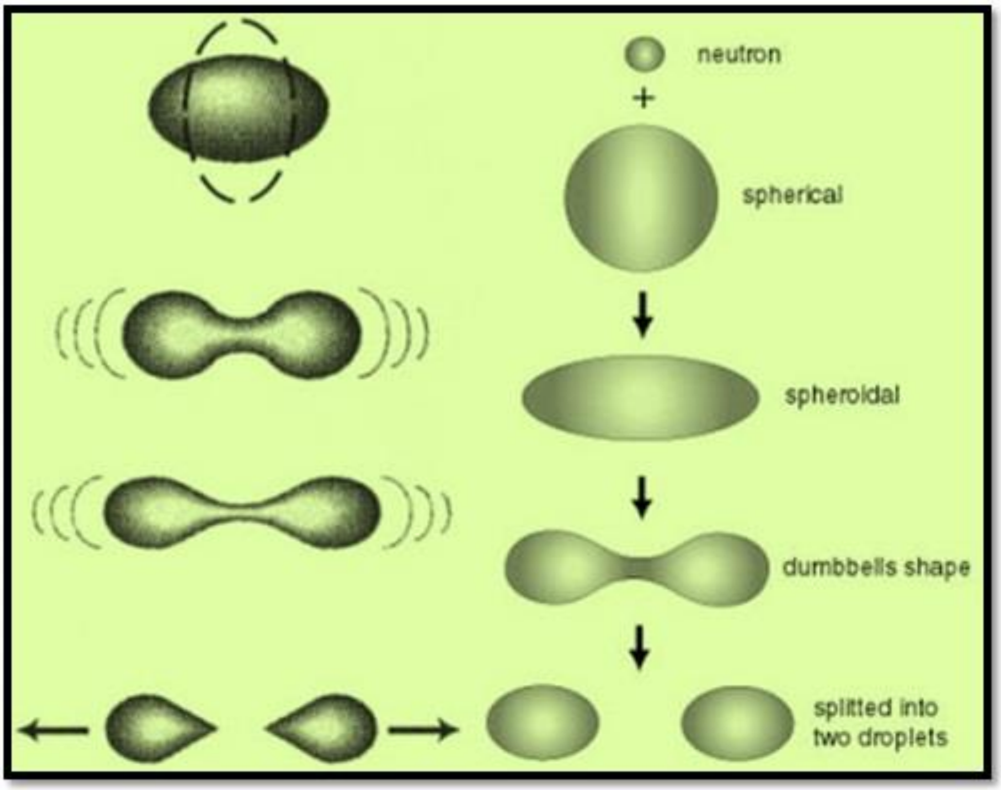
COMPILED & CIRCULATED BY
Dr. Tapanendu Kamilya
Assistant Professor, Department of Physics, Narajole Raj College

Topic:

Elements of Modern Physics (5): Liquid Drop Model: Semi-empirical Mass Formula and Binding Energy, Nuclear Shell Model and Magic Numbers.

Elements of Modern Physics (5):

Liquid Drop Model:



The similar properties between a nucleus and a liquid drop helps to explain nuclear phenomena such as the energetic of nuclear fission and the binding energy of nuclear ground levels. The nucleus has very low compressibility and well-defined surface that idea of considered the nucleus as a liquid drop. The difference as compared with liquid is that the nucleons obey Fermi statistics as well asthe nucleus is a quantum fluid.

Similarities between Nucleus and Liquid Drop:

(i) The liquid drop and nucleus both possess constant density.

*(ii)*The constant binding energy per nucleon of a nucleus is similar to the latent heat of vaporization of liquid.

(iii) The evaporation of a drop corresponds to the radioactive property of nuclei.

(iv) The condensation of drops corresponds to the formation of compound nucleus.

Difference between Nucleus and Liquid Drop:

(i) There is positive charge (proton) present in the nucleus but not found in liquid drop.

*(ii)*Nucleus contains two components like proton and neutron but not in liquid drop.

(iii) The number of nucleons in the heaviest stable nucleus is much smaller than the molecules in an average liquid drop.

Semi-empirical Mass Formula:

Weizsacker in 1935 proposed the following semi-emperical formula to achieve the quantitative and basic understanding of the nuclear binding energy (B.E)

$$B.E. = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{\frac{3}{4}}}$$

Here the constant $a_v = 14.1\ MeV,\ a_s = 13\ MeV,\ a_c = 0.60\ MeV,\ a_n = 19.0\ MeV\ and$

$\delta = 33.5\ MeV\ (for\ even - even\ or\ odd - odd\ nuclei),$

$\delta = 0\ (for\ even - odd\ nuclei)$

(i) *The Volume Energy Term-*

The first term $B_v = a_v A$ represent the volume energy of all nucleons.

We know that $R \propto A^{\frac{1}{3}}$ and $R^3 \propto A$

Therefore, $V = \frac{4}{3}\pi R^3 \propto A$

Therefore, $B_v \propto A$

Hence, $B_v = a_v A$,where $a_v = volume\ coefficient$

(ii) The SurfaceEnergy Term-

Taking nucleus to be spherical like liquid drop its surface area would be $S = 4\pi R^2$

We know that $R \propto A^{\frac{1}{3}}$ and $R^2 \propto A^{\frac{2}{3}}$

Therefore, $B_s \propto A^{\frac{2}{3}}$

Hence, $B_s = -a_s A^{\frac{2}{3}}$, where $a_s = surface\ coefficient$

(iii) The CoulombEnergy Term-

The third term, B_c is the Coulomb electrostatic repulsion between the charged particles, protons, in the nucleus. Since, each charged particle repulses all other charged particles, this term would be proportional to the $\frac{Z(Z-1)}{2}$

Therefore, the energy associated with Coulomb repulsion-

$$B_c = -k\frac{Z(Z-1)}{R}$$

We know that $R \propto A^{\frac{1}{3}}$

$B_c = -a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}}$, where

$R\ is\ replaced\ by\ R_0 A^{\frac{1}{3}}\ and\ a_s = Coulomb\ Energy\ coefficient$

(iv) The AsymmetryEnergy Term-

The fourth term, B_a originates from the asymmetry between the number of protons and neutrons in the nucleus. The asymmetry energy B_a, is directly proportional to the

(a) The number of excess neutron, i.e. $(N-Z) or\ (A-2Z)$
(b) The fraction of nuclear volume in which the excess neutrons are present.

As the nuclear volume $\propto A$, the fractional volume of the nucleus in which excess neutrons are present will be proportional to $\frac{(N-Z)}{A}$

Therefore, $B_a \propto (N-Z)$

$$B_a \propto (N - Z)/A$$

$$B_a = -\, a_n \frac{(N - Z)^2}{A}$$

$$B_a = -\, a_n \frac{(A - 2Z)^2}{A}, where\ a_n = asymmetry\ coefficient$$

(v) The PairingEnergy Term-

The pairing energy $B_a = \pm \frac{\delta}{A^{\frac{3}{4}}}$

Summing up, we get, $B.E. = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{\frac{3}{4}}}$

$$f_B = \frac{B.E}{A} = \{a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{\frac{3}{4}}}\}/A$$

Application:

I. Semi-empirical mass formula helps for prediction of stability of nuclei against β-decay.

II. The stability limit of spontaneous fission can be explained by Semi-empirical mass formula.

Shell Model:

The shell model is partly analogous to the atomic shell model, in that a filled shell results in greater stability. The observation is that there are certain magic numbers of nucleons (2, 8, 20, 28, 50, 82 and 126) which are more tightly bound than the next higher number. This is the origin of the shell model.

The shells for protons and for neutrons are independent of each other. Therefore, the "magic nuclei" exist in which one nucleon type or the other is at a magic number, and "doubly magic nuclei", where both are.

For some variations in the orbital filling, the upper magic numbers are 126 and, hypothetically, 184 for neutrons but only 114 for protons is called island of stability.

Some semi-magic numbers have been found, notably Z = 40 giving nuclear shell filling for the various elements; 16 may also be a magic number.

COMPILED & CIRCULATED BY
Dr. Tapanendu Kamilya
Assistant Professor, Department of Physics, Narajole Raj College

In shell model each nucleon is considered as a single particle that moves independently of others in the time average field of the remaining $(A-1)$ nucleons acting as a core.

Therefore, Schrödinger equation of each nucleons become the harmonic oscillator model. The quantum mechanical rule for $j = l + \frac{1}{2}$ and $j = l - \frac{1}{2}$

According to model, the first six shells are:

Level 0: 2 states (ℓ = 0) = 2.

Level 1: 6 states (ℓ = 1) = 6.

Level 2: 2 states (ℓ = 0) + 10 states (ℓ = 2) = 12.

Level 3: 6 states (ℓ = 1) + 14 states (ℓ = 3) = 20.

Level 4: 2 states (ℓ = 0) + 10 states (ℓ = 2) + 18 states (ℓ = 4) = 30.

Level 5: 6 states (ℓ = 1) + 14 states (ℓ = 3) + 22 states (ℓ = 5) = 42.

Where, for every ℓ there are 2ℓ+1 different values of m_l and 2 values of m_s, which gives a total of 4ℓ+2 states for every specific level.

Taking spin-orbit interaction into calculation, we get-

Level 0 (n = 0): 2 states (j = $\frac{1}{2}$). Even parity.

Level 1 (n = 1): 2 states (j = $\frac{1}{2}$) + 4 states (j = 3⁄2) = 6. Odd parity.

Level 2 (n = 2): 2 states (j = $\frac{1}{2}$) + 4 states (j = 3⁄2) + 6 states (j = 5⁄2) = 12. Even parity.

Level 3 (n = 3): 2 states (j = $\frac{1}{2}$) + 4 states (j = 3⁄2) + 6 states (j = 5⁄2) + 8 states (j = 7⁄2) = 20. Odd parity.

Level 4 (n = 4): 2 states (j = 1⁄2) + 4 states (j = 3⁄2) + 6 states (j = 5⁄2) + 8 states (j = 7⁄2) + 10 states (j = 9⁄2) = 30. Even parity.

Level 5 (n = 5): 2 states (j = 1⁄2) + 4 states (j = 3⁄2) + 6 states (j = 5⁄2) + 8 states (j = 7⁄2) + 10 states (j = 9⁄2) + 12 states (j = 11⁄2) = 42. Odd parity.

Where, for every j there are $2j + 1$ different states from different values of m_j.

The quantum number n, l, j and the designation of energy states in order of the energy in accordance with the above notational scheme is shown in table below, up to magic number 126. The thick horizontal lines signal the closure of a shell, where marked changes occur in the nucleonic binding energy.

COMPILED & CIRCULATED BY
Dr. Tapanendu Kamilya
Assistant Professor, Department of Physics, Narajole Raj College

Table 8.2 : Nucleonic sub-shells and shells

n	l	j	Designation and no. of p or n to fill sub-levels		Total number of p or n in closed shells
1	0	1/2	$(1s_{1/2})^2$	2	2
1	1	3/2	$(1p_{3/2})^4$		
1	1	1/2	$(1p_{1/2})^2$	6	8
1	2	5/2	$(1d_{5/2})^6$		
2	0	1/2	$(2s_{1/2})^2$		
1	2	3/2	$(1d_{3/2})^4$	12	20
1	3	7/2	$(1f_{7/2})^8$		
2	1	3/2	$(2p_{3/2})^4$		
1	3	5/2	$(1f_{5/2})^6$		
2	1	1/2	$(2p_{1/2})^2$		
1	4	9/2	$(1g_{9/2})^{10}$	30	50
1	4	7/2	$(1g_{7/2})^8$		
2	2	5/2	$(2d_{5/2})^6$		
2	2	3/2	$(2d_{3/2})^4$		
3	0	1/2	$(3s_{1/2})^2$		
1	5	11/2	$(1h_{11/2})^{12}$	32	82
1	5	9/2	$(1h_{9/2})^{10}$		
2	3	7/2	$(2f_{7/2})^8$		
2	3	5/2	$(2f_{5/2})^6$		
3	1	3/2	$(3p_{3/2})^4$		
3	1	1/2	$(3p_{1/2})^2$		
1	6	13/2	$(1i_{13/2})^{14}$	44	126

COMPILED & CIRCULATED BY
Dr. Tapanendu Kamilya
Assistant Professor, Department of Physics, Narajole Raj College

Frequently Asked Questions/Numerical:

For theoretical questions and problems in this section students can solve the problems of Modern Atomic and Nuclear Physics, Author- A.B. Gupta, Published by Books & Allied Pvt. Ltd. (2017 Ed.).

Example 1:Using the semi-empirical binding energy formula, calculate the binding energy of Ca_{20}^{40}.

Example 2:Predict (i) the ground state spins and (ii) the parities of $Al_{13}^{27}, Ar_{18}^{41}$.

Example 3:Find the total angular momentum and parity for ground stateof S_{16}^{33}.

References:

(i) *Modern Atomic and Nuclear Physics, Author- A.B. Gupta, Published by Books & Allied Pvt. Ltd. (2017 Ed.).*
(ii) https://en.wikipedia.org/wiki/Nuclear_shell_model

Link to Audio visual Lectures (e-Lectures) on this topic given by Distinguish Professors of Indian &Foreign Universities:

(1) https://www.youtube.com/watch?v=3bwcXPmF2VA
(2) https://www.youtube.com/watch?v=8vMwzkOi0v4
(3) https://www.youtube.com/watch?v=rUU_1yUPaus
(4) https://www.youtube.com/watch?v=Rd0CJje59bE