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C9T (Elements of Modern Physics) , Topic :- Heisenberg's Uncertainty Principle(Unit2)

- **Heisenberg's Uncertainty Principle :** For a particle of atom's magnitude in motion , it is impossible to determine both the position and the momentum simultaneously with perfect accuracy.

According to Heisenberg uncertainty principle, the product of uncertainty in position co-ordinate in the x-direction (Δx) and the uncertainty momentum along the same direction (Δp_x) at the same instant, is the order of $\hbar$ $(\frac{h}{2\pi})$.

$$\text{i.e.} \Delta x . \Delta p_x \geq \hbar$$

Along three directions, uncertainty relations are

$$\Delta x . \Delta p_x \geq \hbar$$

$$\Delta y . \Delta p_y \geq \hbar$$

$$\Delta z . \Delta p_z \geq \hbar$$

- **The exact statement of uncertainty principle :**

The product of the uncertainties in determing the position and momentum of the particle can be never be smaller than the number order $\frac{1}{2}\hbar$

$$\text{i.e. } \Delta q . \Delta p \geq \frac{1}{2}\hbar$$

where Δp is the in uncertainty in determing the momentum.

where Δq is the in uncertainty in determing the position.

- Other relations (Uncertainty) are :
  1. Energy-time relation : $\Delta E . \Delta t \geq \frac{\hbar}{2}$

     where ΔE is the in uncertainty in determing the energy.

     where Δt is the in uncertainty in determing the time.
  2. Angular momentum- angle relation : $\Delta J \Delta \theta \geq \frac{\hbar}{2}$

where ΔJ is the in uncertainty in determing the angular-momentum.

where $\Delta\theta$ is the in uncertainty in determing the angle.

- **Proof of Heisenberg uncertainty relation by Gamma-ray microscope experiment:**

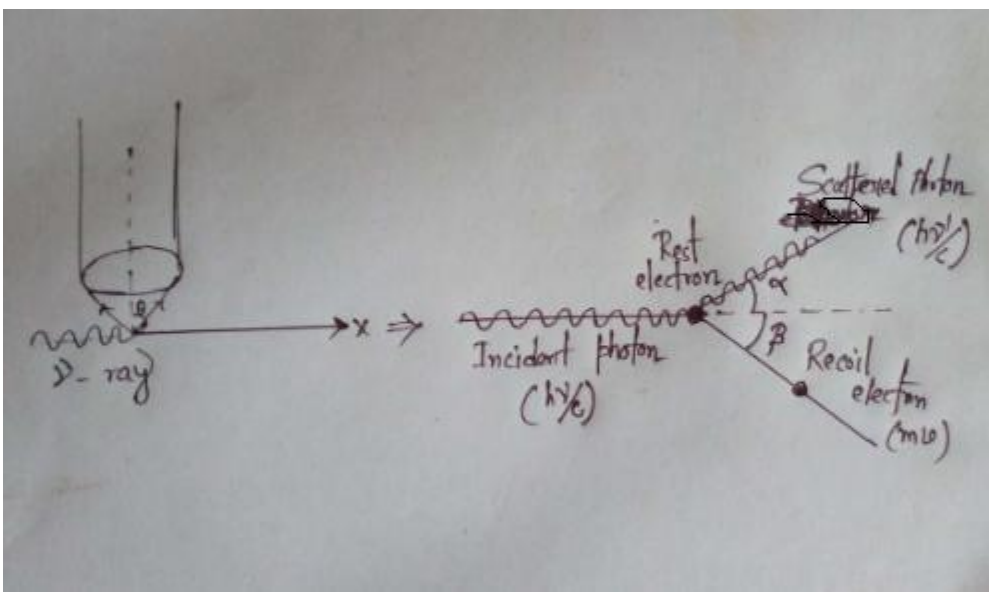
  The position of an electron is determined by a very high resolving power microscope in which the electron is illuminated by γ- rays.

  The limit of resolution of the microscope is , $\Delta x = \frac{\lambda}{2Sin\theta}$ …………………(1)

  Where Δx represents the uncertainty in determing the position of the electron, λ is the wavelength of illuminating γ- rays and θ is the angle of resolution.

  When γ- rays incidents on the electron strikes the electron and suffers Compton effect. i.e. we get scattered photon and recoil electron.

- **Calculation :** Let a photon of momentum $\frac{h\nu}{c}$ strikes an electron which is initially at rest. The striking photon transfer a momentum *mv* to the electron and the photon is scattered into the microscope.



According to the law of conservation of momentum along the direction of incident photon ,

$$\frac{h\nu}{C} + 0 = \frac{h\nu'}{C} Cos\alpha + mvCos\beta$$

$$\Rightarrow \; mvCos\beta = \frac{h}{C}(\nu - \nu' \; Cos\alpha) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\text{....(2)}$$

Since, α varies (within the microscope) clearly from $(90^0 - \theta)$ to $(90^0 + \theta)$, so the spread in *x* component of momentum is given

$$\frac{h}{C}[\nu - \nu' Cos(90^0 - \theta)] \leq \; p_x \leq \frac{h}{C}[\nu - \nu' Cos(90^0 + \theta)]$$

$$\Rightarrow \frac{h}{C}(\nu - \nu' Sin\theta) \leq \; p_x \leq \frac{h}{C}(\nu + \nu' Sin\,\theta)$$

Therefore the uncertainty in momentum is given by ,

$$\Delta p_x = \frac{h}{C}(\nu + \nu' Sin\,\theta) - \frac{h}{C}(\nu - \nu' Sin\theta)$$

$$= \; \frac{2h}{C}\nu' Sin\,\theta$$

$$\approx \frac{2h}{\lambda} \; Sin\,\theta \ldots\ldots\ldots\ldots\ldots\ldots \text{ (3)}$$

Multiplying equation (1) and (3), $\Delta x . \Delta p_x = \frac{\lambda}{2Sin\theta} . \frac{2hSin\theta}{\lambda}$

i.e. $\Delta x . \Delta p_x \approx h$

i.e. $\boldsymbol{\Delta x . \Delta p_x \geq \frac{\hbar}{2}}$

.: Heisenberg’s uncertainty relation is proved.

- **Physical significance of the Heisenberg's uncertainty principle :**

Heisenberg's uncertainty relation $\Delta x . \Delta p_x \approx \hbar$ leads the following conclusions.

(1) If the $\Delta p_x = 0$ then $\Delta x \rightarrow \infty$ i.e. if the position of a particle in motion is determine accurately by an experiment $(\Delta x = 0)$ the momentum of the particle can not be measured i.e. $\Delta p_x \rightarrow \infty$.

(2) For a heavy particle $\hbar/m$ very small thus $\Delta x . \Delta v$ is very small. Thus both the position and the velocity can be determine more accurately if the particle is heavy. For a very very heavy particle $\frac{\hbar}{m} \rightarrow 0$, thus both the position and the velocity can be perfectly determined.

REVIEW QUESTIONS AND PROBLEMS

1. A pion and a proton can briefly join together to form a delta particle. A measurement of the energy of the system shows a peak at 1236 MeV, corresponding to the rest energy of the delta particle, with an experimental spread of 120 MeV. What is the lifetime of the delta particle.
2. In the ground state of hydrogen, the uncertainty in the position of the electron is roughly 0.10 nm. If the spread of the electron is approximately the same as the uncertainty in its speed, about how fast is it moving?
3. What is the uncertainty in the position of an electron when its velocity is known to 5% the speed of light?
4. If the maximum velocity v of an electron in copper wire is about 2.0×10^8 cm/s, with an uncertainty of 1% how accurately can the electron's position be determined?
5. What is the linear size of the smallest box in which you can confine an electron if you want to know for certain that the electron's speed is no more than 13 m/s?