Prof. Surajit Dhara

Guest Teacher,

Dept. Of Physics, Narajole Raj College

C9T (Elements of Modern Physics) , Topic :- Matter waves (Unit 2)

❖ Introduction :

The physical phenomena like Compton effect, Zeeman effect can be explained on the basis of electromagnetic radiation must be considered corpuscular in character. There are also phenomena like interference, diffraction which can be explained on the basis of wave nature of light.

Electromagnetic radiation therefore must be considered as a wave in some process and a particle in others. Acceptance of dual nature of light can't be reading acceptable because of the apparently contradictory aspects of the two nature.

- **Matter Wave :** Lous De – Broglie argued that the wave character is also associated with all particles in motion and wavelength of this wave $\lambda = \frac{h}{p}$. This wave is called matter wave. This is also known as De- Broglie's hypothesis.

- **Deviation of wavelength of matter wave :**

According to Planck, energy associated with photon, $E = h\nu$ …………(1)

[where $h \Rightarrow Planck's\ sconstant$ $\nu \Rightarrow$ frequency of matter wave]

Considering particle nature alone, energy of the radiation ,

$E = \sqrt{(p^2C^2 + m_0^2C^4)}$ taking raletivistic case.

As must mass of photon is zero i.e. $m_0 = 0$

Thus, $E = pC$ ………..(2)

Compairing equ. (1) and (2) , we get,

$$E = h\nu$$

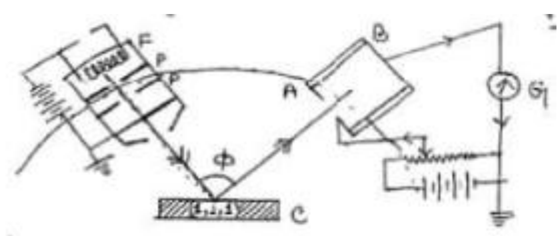
$$\Rightarrow p = \frac{h\nu}{c}$$

$$\boldsymbol{\Rightarrow p = \frac{h}{\lambda}}$$

Where $\lambda = \frac{c}{\nu}$ is the wavelength of electromagnetic radiation.

According to De- Broglie , the wave character is also associated with all particles in matter , whose wavelength $\boldsymbol{\lambda = \frac{h}{p}}$ **.** This is ***De-Broglie hypothesis***.

- **Davission and Germer's Experiment :**



- **Apparatus :** F is the tungsten filament which emits electron when it is heated.(P,P) is the having narrow hole S.It is maintained at a position voltage V w.r. to filament F, so that the electrons are accelerated by p.d. C is the nickel crystal with its plane is placed normal to the beam of electrons. When electron beam is incident on the crystal, electrons are scaled in all directions by the atom of the crystal.

  AB is the character in which the electron beams are received. The chamber can rotate about an axis in the face of C passing through the point of interference of the electron beam. The current is measured by the galvanometer G. The apparatus is enclosed in a evacuated chamber.

➢ **Method :** P.D is applied between the filament F and the metal plate P. The chamber is set at different angles and for each setting of the chamber , the current is noted. The current is directly proportional to the number of scattered electrons entering the chamber per second.

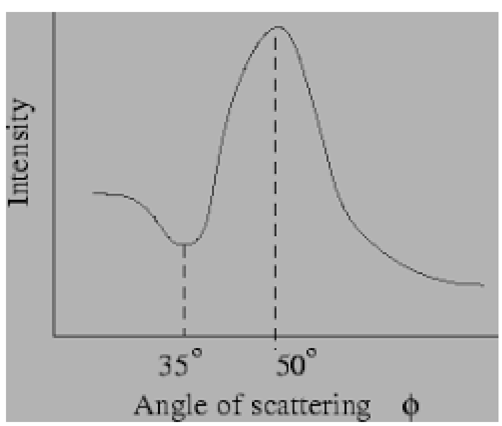
Thus the intensity of the scattered beam is measured as a function of the angle scattering. The graph shows the intensity verses angle of scattered beam at constant p.d. 54 volt. It is observed that the intensity is maximum a $\phi = 50^0$ when V = 54 volt.

- <u>Experimental value of λ</u> :
  Bragg's Law, $dsin\theta = n\lambda$, For 1st order n = 1, d = interplaner spacing = $2.15A^0$,
  .: λ = $2.15\ X\ sin50^0 A^0 = 1.65\ A^0$.

- <u>Theoritical Value of λ</u> : (From De- Borglie Relation)

$$\boldsymbol{\lambda = \frac{h}{\sqrt{2mqV}} = \frac{12.28}{\sqrt{V}} = \frac{12.28}{\sqrt{54}} = 1.67\ A^0}$$

Thus we see thst experiment value is nearly equal to theoretical value.



- **Phase Velocity :** When a single wave of a definite wavelength travels in a medium , its velocity of propagation in the medium is called the " ***phase velocity*** or ***wave velocity*** ".

- **Group Velocity :** If a number of waves of different wave lengths are moving with different velocities in a medium, then the observed velocity is the velocity of the wave packet formed by the the waves. This is called the " ***group velocity*** ".

- **Derive a relation between group velocity and phase velocity for a wave representing a free particle :**

Let us consider a group formed by the following two waves of same amplitude $\boldsymbol{a}$ but slightly different frequencies and wavelengths :

$$y_1 = a\,\text{Sin}(\omega t - kx)$$

$$y_2 = a\,\text{Sin}\,[(\omega + d\omega)t - (k + dk)x]$$

Where $k = \frac{2\pi}{\lambda}$ is called the propagation constant.

The resultant displacement: $y = y_1 + y_2$

$$\therefore y = a\,\text{Sin}(\omega t - kx) + a\,\text{Sin}\,[(\omega + d\omega)t - (k + dk)x$$

$$= 2a\,\text{Sin}[\,(\omega + \frac{d\omega}{2})t - \left(k + \frac{dk}{2}\right)x].\,Cos\,\frac{d\omega.t - dk.x}{2}$$

As $\omega + \frac{d\omega}{2} \approx \omega$ and $k + \frac{dk}{2} \approx k$

$$= 2a\,\text{Sin}(\omega t - kx)\,\text{Cos}\,(\frac{d\omega}{2}t - \frac{dk}{2}x)$$

$$= 2a\,\text{Cos}\,(\frac{d\omega}{2}t - \frac{dk}{2}x)\,\text{Sin}(\omega t - kx)$$

$$\boxed{\boldsymbol{y = A.Sin\,(\omega t - kx)}}$$

Where $A = 2a\,\text{Cos}\,(\frac{d\omega}{2}t - \frac{dk}{2}x)$

The sine factor represents : The wave of same frequency and same propagation constant k as one of the component of the waves having velocity, $\boldsymbol{v_p = \frac{\omega}{k}}$ **;** called ***phase velocity*** .

The cosine factor represents : A slowly varying function both x and t *and* modulates the amplitude of the resultant wave, whose velocity $\boldsymbol{v_g = \frac{d\omega}{dk}}$ **;** called ***group velocity*** .

- **Relation :**

$$\textbf{Phase velocity, } \boldsymbol{v_p = \frac{\omega}{k}}$$

$$\textbf{Group velocity, } \boldsymbol{v_g = \frac{d\omega}{dk}}$$

Putting $\boldsymbol{k = \frac{2\pi}{\lambda}}$**, we get** $\boldsymbol{v_g = \frac{d\omega}{d(\frac{2\pi}{\lambda})}}$

$$\boldsymbol{= -\frac{\lambda^2}{2\pi}\frac{d}{d\lambda}(\frac{2\pi\nu}{\lambda})}$$

$$\boldsymbol{= -\lambda^2(-\frac{\nu}{\lambda^2} + \frac{1}{\lambda}\frac{d\nu}{d\lambda})}$$

$$\boxed{\boldsymbol{v_g = v_p - \lambda\frac{dv_p}{d\lambda}}}$$

* In a non-dispersive medium $\frac{dv_p}{d\lambda} = 0$

$$v_g = v_p$$

REVIEW QUESTIONS

1. Show that the particle velocity is equal to the group velocity of a wave packet.
2. Show that, for a non-ralativistic free particle the phase velocity is half of the group velocity.

PROBLEMS

1. What is the de- Broglie wavelength of an electron kinetic energy is 200 eV.
2. Calculate the de- Broglie wavelength of an electron passing through a P.D. of 150 volts.
3. Find the de- Broglie wavelength of 1 MeV proton: mass of proton = 1.67×10^{-27} Kg.
4. The most rapidly moving valence electron is metallic sodium, at the absolute zero temperature has a kinetic energy 3 eV. Show that its de- Broglie wavelength is 7 A^0.