Prof. Surajit Dhara

Guest Teacher,

Dept. Of Physics, Narajole Raj College

C14T (Statistical Mechanics) , Topic :-Applications of BE-statistics

❖ Planck's radiation law (Statistics of photon gas):

Let a black body chamber of volume *V* at temperature *T* K be filled with radiant energy that can be considered as a 'gas' whose particles are photons having unit spin angular momentum. Hence they are bosons and we shall use BE-distribution to derive the law of black body radiation.

The number of photons with energy between *E* and *E + dE* is

$$N(E)dE = \frac{g(E)}{e^{\alpha}e^{E/kT}-1}dE \qquad \text{..........(1)}$$

$g(E)dE$ being the no. of quantum states of photons of energy between *E* and *E + dE.*

The no of quantum states corresponding to momentum between *p* and *p+dp* is

$$g(p)dp = \frac{V}{h^3}4\pi p^2 dp$$

Since each photon is endowed with unit spin angular momentum, there are two allowed momentum states for photon. So for photons

$$\boldsymbol{g(p)dp = \frac{8V}{h^3}\pi p^2 dp} \quad \textbf{......(2)}$$

Now the energy of a photon of frequency *v* is $E = hv$ .So its momentum $p = hv/c$ and $dp = \left(\frac{h}{c}\right)dv$.

Substituting these values of *p* and *dp* in eqn.(2) ,we obtained the number of quantum states with frequency between *v* and *v+dv* as :

$$\boldsymbol{g(v)dv = \frac{8\pi V(\frac{h}{c})(\frac{hv}{c})^2}{h^3} dv = \frac{8\pi V}{c^3} v^2 dv} \quad \textbf{.....(3)}$$

In a black body chamber at constant temperature , photons of different energy are ***absorbed*** and ***re-emitted*** by the walls of the enclosure i.e. the number of photons in the system is not constant.

$$\boldsymbol{\sum \partial N_i \neq 0}$$

Which can be taken care of by setting the Lagrange multiplier, $-\alpha = 0$

So, expressing eqn.(1) in terms of *v* and setting α = 0 , we obtained , using eqn.(2)

$$N(v)dv = \frac{g(v)}{e^{hv/kT}-1}dv = \frac{8\pi h}{c^3}\frac{v^2}{e^{hv/kT}-1}dv \quad ......(4)$$

Which gives the number of photons of frequency between *v* and *v+dv* in the enclosure of volume *V* and at a temperature *T*.

Since the energy of a photon of frequency *v* is *hv* , the energy per unit volume of the enclosure to the ***energy density u(v)***, within a frequency range (bandwidth) ***dv*** is

$$u(v) = \frac{hv}{V}N(v) = \frac{8\pi h}{c^3}\frac{v^3}{e^{hv/kT}-1}$$

Which is the famous ***Planck's law*** of radiation in terms of frequency ***v*** .

- Planck's law of radiation in terms of wavelength λ and other deductions there from such as Wien's law ,Rayleigh-Jeans law ,Stefan –Boltzmann law etc. have all been discussed thoroughly in chapter ***Thermal Radiation.***