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Topic:

Matrix: Addition and Multiplication of Matrices. Null Matrices. Diagonal, Scalar and Unit Matrices, Upper-Triangular and Lower-Triangular Matrices, Transpose of a Matrix, Symmetric and Skew-Symmetric Matrices, Conjugate of a Matrix, Hermitian and Skew-Hermitian Matrices, Singular and Non-Singular matrices, Orthogonal and Unitary Matrices, Trace of a Matrix.

MATRICES

1. Objective &Relevance of the Section:

The arrangement of a system of numbers mn in rectangular form in m- rows along with n- columns and finally bounded by a [ ] bracket is called a **Matrix (plural form matrices)**.Matrices are used in the study of electrical circuits, quantum mechanics, optics,solving of differential equations, etc. A matrix with the same number of rows and columns, sometimes used to represent a linear transformation from a vector space to itself, such as reflection, rotation, or shearing.

Example: $[A]=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$

In this section, our main objective isto learn the Addition and Multiplication of Matrices. Null Matrices. Diagonal, Scalar and Unit Matrices, Upper-Triangular and Lower-Triangular Matrices, Transpose of a Matrix, Symmetric and Skew-Symmetric Matrices, Conjugate of a Matrix,Hermitian and Skew- Hermitian Matrices, Singular and Non-Singular matrices, Orthogonal and Unitary Matrices, Trace of a Matrix as per CBCS syllabus of C8T of Physics (H) under Vidyasagar University.

2. Concept of different Matrices:

Row Matrix: $[A]=[1 \quad 2 \quad 3]$

Column Matrix: $[A]=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$

Null Matrix: $[A]=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

A matrix with all elements are zero is called a null matrix.

Diagonal Matrix: $[A]=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$

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A square matrix with all the non-diagonal elements are zero is called a diagonal matrix. If all the diagonal elements are "1". Then, the matrix is called Unit or Identity Matrix.

Unit or Identity Matrix: $[A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Singular Matrix: $[A] = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

A square matrix is called singular matrix if its determinant det A is zero. If ,det A is not zero the matrix [A] is called non-singular matrix.

Non-Singular Matrix: $[A] = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 6 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

Upper Triangular Matrix: $[A] = \begin{bmatrix} 2 & 2 & 3 \\ 0 & 6 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

If all the elements of a matrix [A] below the leading diagonal of a square matrix are zero, then it is called an upper triangular matrix.

Lower Triangular Matrix: $[A] = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 6 & 0 \\ 3 & 4 & 5 \end{bmatrix}$

If all the elements of a matrix [A] above the leading diagonal of a square matrix are zero, then it is called a lower triangular matrix.

Symmetric Matrix: $[A] = \begin{bmatrix} a & h & g \\ h & p & f \\ g & h & c \end{bmatrix}$

A square matrix is called a symmetric matrix if for all values of I and j, $a_{ij}=a_{ji}$ or A′=A. i.e. After interchanging the row and column we get the same matrix.

Skew-Symmetric Matrix: $[A] = \begin{bmatrix} 0 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{bmatrix}$

A square matrix is called a symmetric matrix if for all values of I and j, $a_{ij}=-a_{ji}$ or A′=-A.

Transpose of Matrix:

If the rows and the corresponding columns of a matrix A are interchanged, then the resulting matrix is called the transpose of the matrix A and denoted by A′.

$[A] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $[A]' = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

If A′=A; then the matrix is called symmetric matrix. Otherwise, it A′=-A; then the matrix is called skew symmetric matrix.

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Orthogonal Matrix:

A square matrix A is said to be orthogonal if, A′A=I.

$[A] = \begin{bmatrix} Cosa & -Sina \\ Sina & Cosa \end{bmatrix}$ Therefore, $[A]' = \begin{bmatrix} Cosa & Sina \\ -Sina & Cosa \end{bmatrix}$

Then, $[A]'[A] = \begin{bmatrix} Cosa & Sina \\ -Sina & Cosa \end{bmatrix}\begin{bmatrix} Cosa & -Sina \\ Sina & Cosa \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Conjugate Matrix:

If the elements of a matrix are complex, then, its conjugate is represented by A*

$[A] = \begin{bmatrix} 1+i & 2+3i \\ 1-i & 2-3i \end{bmatrix}$ then $A^* = \begin{bmatrix} 1-i & 2-3i \\ 1+i & 2+3i \end{bmatrix}$

Hermitian Matrix:

A square matrix A = $[a_{ij}]$ is called hermitian, if every ijth element of A is equal to complex conjugate of jith element of A. i.e. $a_{ij} = a_{ij}^*$

$[A] = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ then $[A^*]' = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

Skew-Hermitian Matrix:

A square matrix A = $[a_{ij}]$ is called skew-hermitian, if every ijth element of A is equal to negative conjugate complex of jith element of A. i.e. $a_{ij} = -a_{ij}^*$

$$[A] = \begin{bmatrix} i & 2-3i & 4+5i \\ -2-3i & 0 & 2i \\ -4+5i & 2i & -3i \end{bmatrix}$$

Here, $[A^*]' = -A$

Unitary Matrix: If, $[A^*]'A = I$, then [A] is called Unitary Matrix.

Matrix Addition: $[A] + [B] = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 3 \\ 2 & 2 & 4 \\ 3 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 6 \\ 4 & 5 & 8 \\ 6 & 7 & 7 \end{bmatrix}$

Matrix Addition is commutative: A+B= B+A

Matrix Addition is associative: A+(B+C)= (A+B)+C

Matrix Multiplication:

$[A] \times [B] = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix} = AB$

Trace of Matrix:

Trace means sum of diagonal elements.

$$[A] = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

Trace of [A]= Trace of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} = 9$

The trace of the product of two matrices [A] and [B] is independent of multiplication. i. e. $T_r[AB\} = T_r[BA]$

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3. Summary:

Herein, we learn the Addition and Multiplication of Matrices, Null Matrices. Diagonal, Scalar and Unit Matrices, Upper-Triangular and Lower-Triangular Matrices, Transpose of a Matrix, Symmetric and Skew-Symmetric Matrices, Conjugate of a Matrix, Hermitian and Skew- Hermitian Matrices, Singular and Non-Singular matrices, Orthogonal and Unitary Matrices, Trace of a Matrix 4. 4.

Frequently Asked Questions:

a. Prove that every square matrix can be expressed as a sum of a symmetric matrix and a skew symmetric matrix.

b. Show by means of an example that in matrices A+B=0 does not necessarily mean that either A=0 or B=0, where 0 stands for the null matrix.

c. *Express* = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}$ *as sum of a symmetric and a skew-symmetric matrix.*

d. Prove that $[AB]^n = A^n B^n$

e. $A=\begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ *show that* $U=[I-A][1+A]^{-1}$ *is unitary.*

f. Show that the inverse of a unitary matrix is unitary and the product of two unitary matrices is unitary.

g. Show that $A=\begin{bmatrix} Cos\beta & -Sin\beta \\ Sin\beta & Cos\beta \end{bmatrix}$ *is orthogonal.*

References:

(i) Principles of Mathematical Physics, Author- S. P. Kuila, published by NCBA (2018 Ed.).

(ii) Mathematical Physics: Author-H.K. Dass& Rama Verma,published by S. Chand(2018 Ed.).