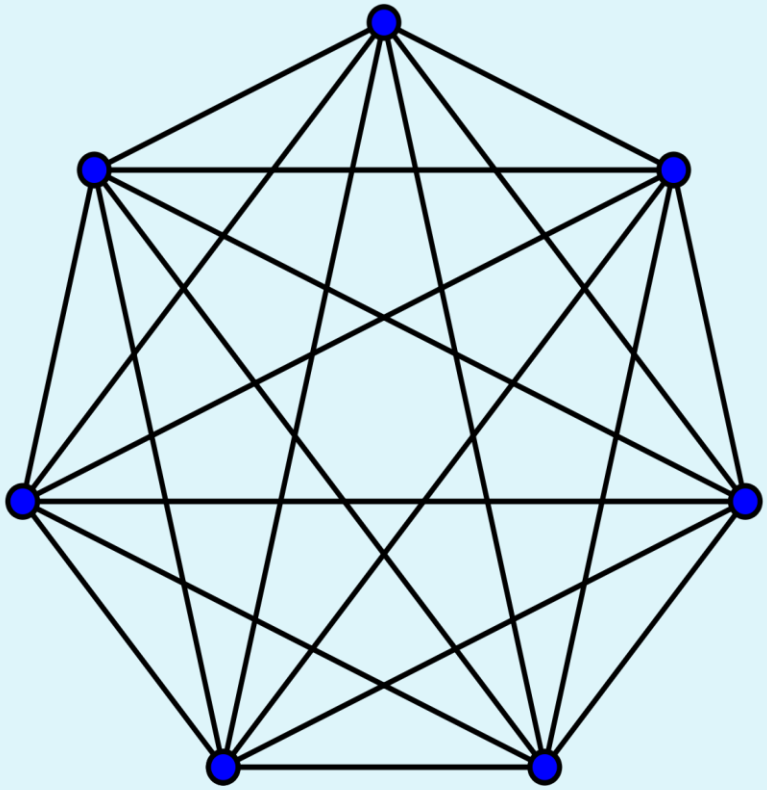


Introduction to Graph Theory

Presented by
Dr. Akul Rana,
Narajole Raj College

Outline

- History of Graph Theory
- Definition
- Graph Terminologies
- Different Types of Graphs

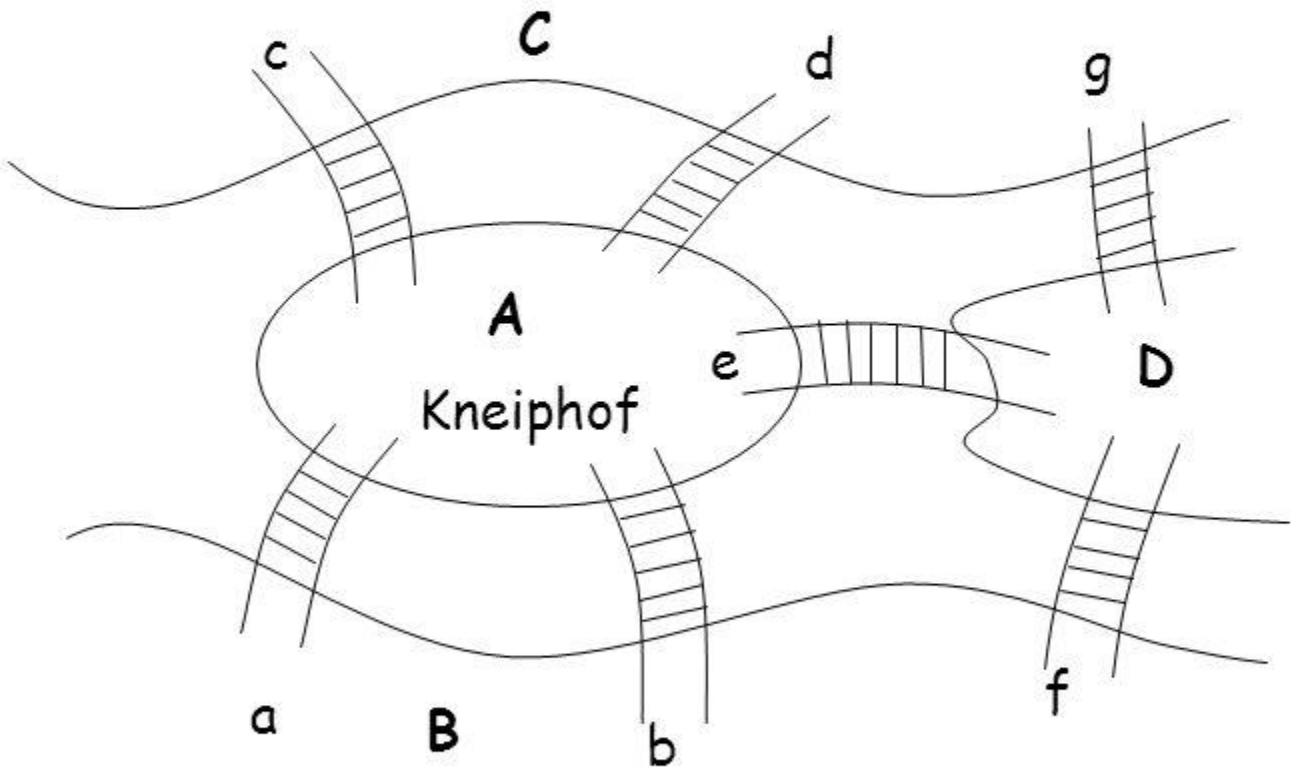
History

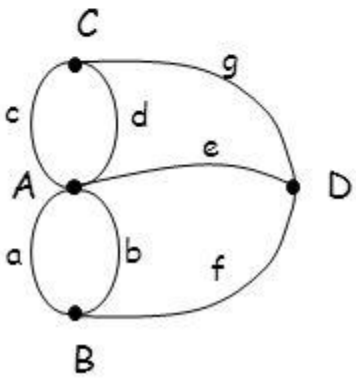
The origin of graph theory can be traced back to Euler's work on the Konigsberg bridge problem (1735) which led to the concept of an Eulerian graph. Then the study of cycles on polyhedra by Thomas P. Kirkman (1806-1895) and William R. Hamilton (1805-1865) led to the concept of Hamiltonian Graphs.

Konigsberg Bridge Problem

- A river Pregel flows around the island Keniphof and then divides into two.
- Four land areas A, B, C, D have this river on their borders.
- The four lands are connected by 7 bridges a – g.
- Determine whether it's possible to walk across all the bridges exactly once in returning back to the starting land area.

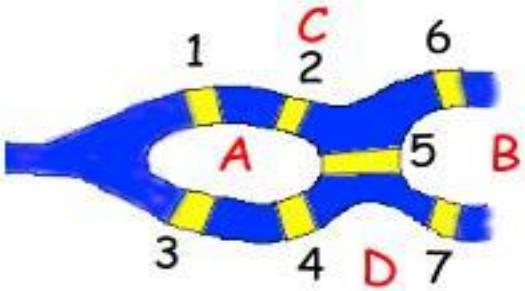
Konigsberg Bridge Problem (Cont.)



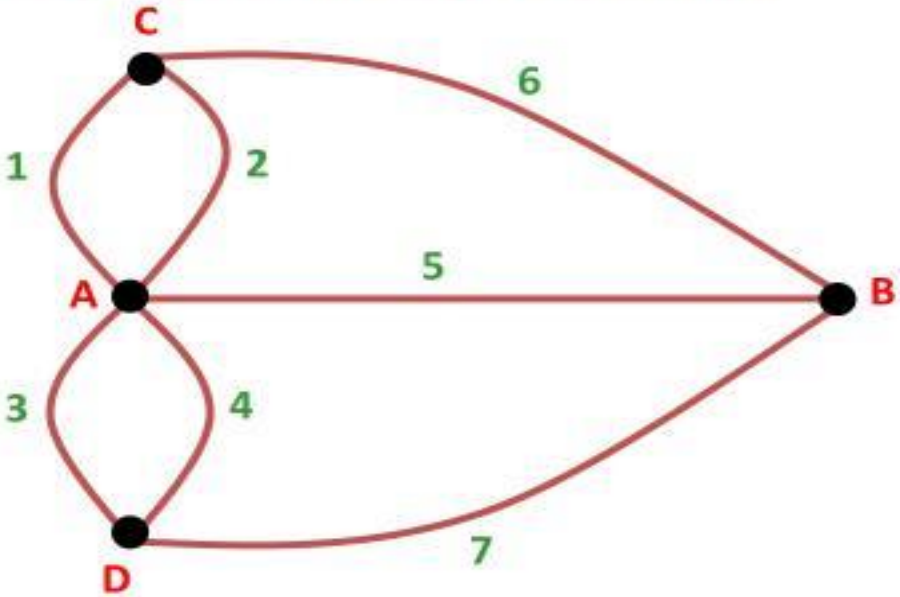


KONIGSBERG BRIDGE

~ a Graph Theory Problem



A graph of the bridge problem:



Edges – (Bridges) A link joining two vertices in a graph.

Vertex (vertices) - A point on a graph where one or more edges end.

Graph – A finite set of dots (vertices) and connecting links (edges). Each edge must connect two different vertices.

Can you draw this graph without lifting your pencil and without redrawing an edge?

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Graph Definitions and Notation

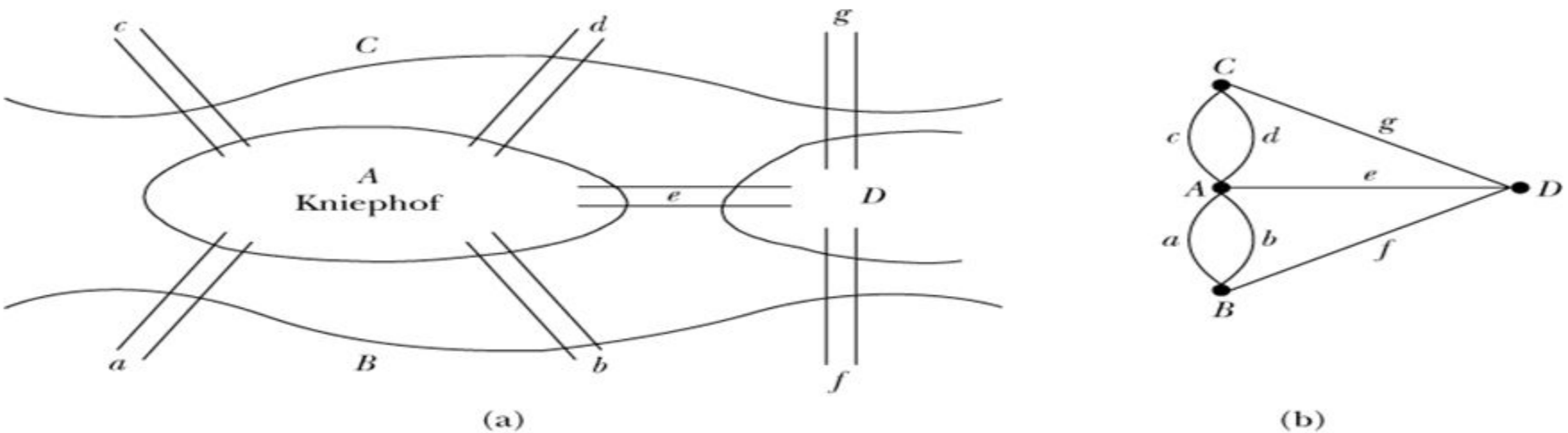

FIGURE 10.1 Königsberg bridge problem

- The Königsberg bridge problem is as follows: Starting at one land area, is it possible to walk across all of the bridges exactly once and return to the starting land area?
- In 1736, Euler represented the Königsberg bridge problem as a graph, as shown in Figure 10.1(b), and answered the question in the negative.

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WHY IS IT IMPOSSIBLE?

- Euler's Solution involved the observation that when a vertex is visited in the middle of the process of tracing a graph, there must be an edge coming into the vertex and another vertex leaving it and so the order of the vertex must be an even number.

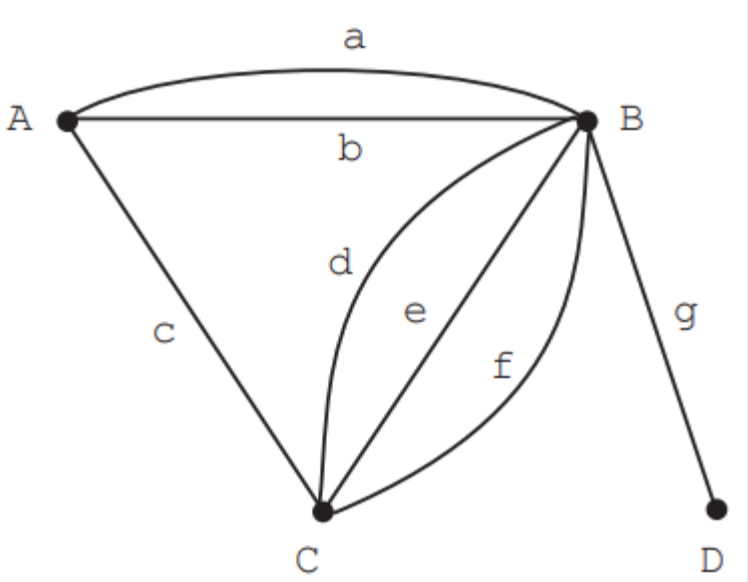
- This must be true for all but at most two of the vertices-the one you start at and so a connected graph is traversable iff it has at most two vertices of odd order.

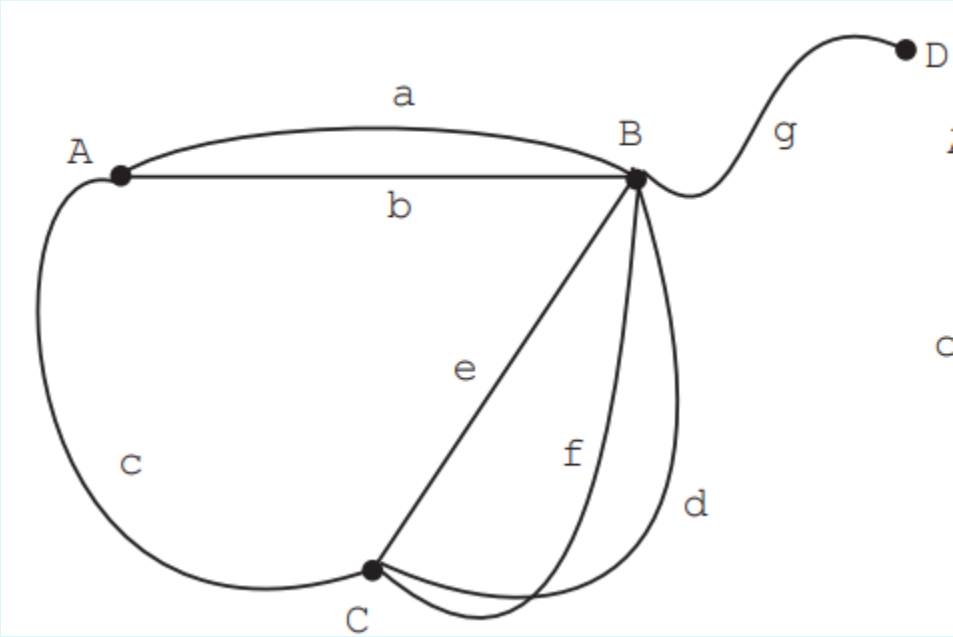
- Now the graph shows that there are more than two vertices of odd order and therefore the graph cannot be traced.

Definition of A Graph

- A graph, G, consists tof two sets, V and E.
  - V is a finite, nonempty set of vertices.
  - E is set of pairs of vertices called edges.
- The vertices of a graph G can be represented as V(G).
- Likewise, the edges of a graph, G, can be represented as E(G).
- Graphs can be either undirected graphs or directed graphs.
- For a undirected graph, a pair of vertices (u, v) or (v, u) represent the same edge.
- For a directed graph, a directed pair <u, v> has u as the tail and the v as the head. Therefore, <u, v> and <v, u> represent different edges.

The following two graphs represents the same graph.





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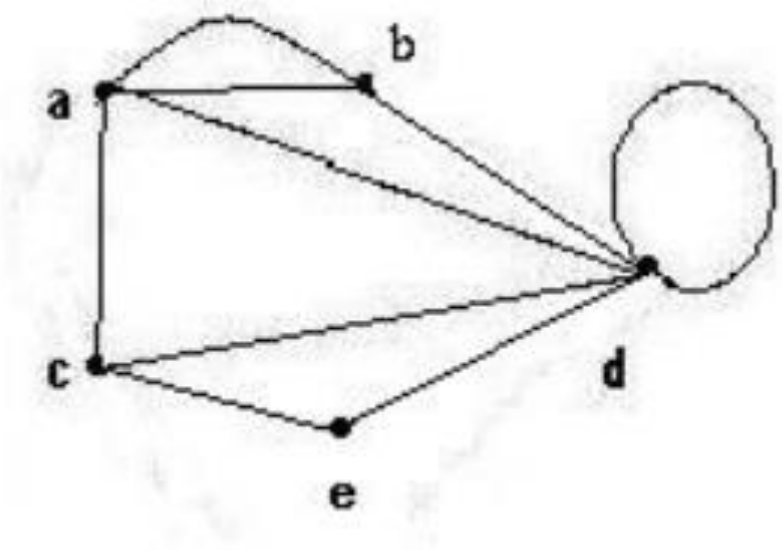
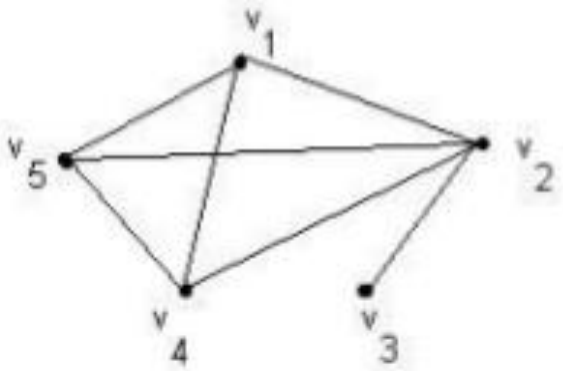
Special edges

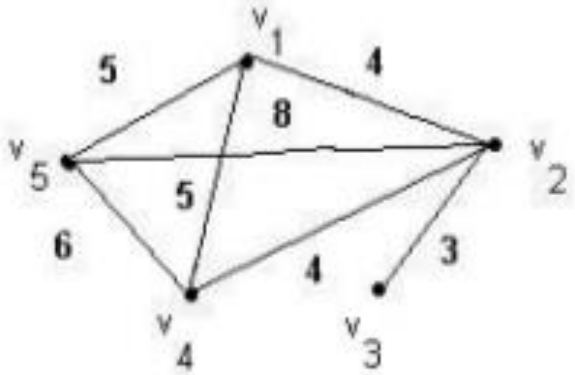
- Parallel edges
  - Two or more edges joining a pair of vertices
    - in the example, **a** and **b** are joined by two parallel edges
- Loops
  - An edge that starts and ends at the same vertex
    - In the example, vertex **d** has a loop



Special graphs

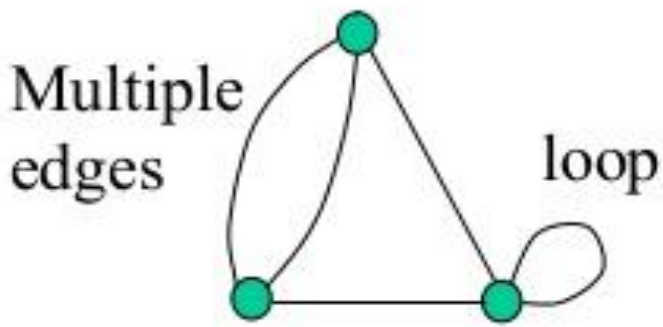
- Simple graph
  - A graph without loops or parallel edges.
- Weighted graph
  - A graph where each edge is assigned a numerical label or "weight".



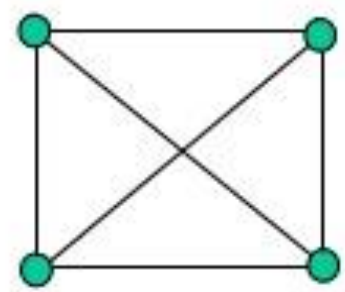


Simple Graph

- *Simple graph* : A graph has no loops or multiple edges



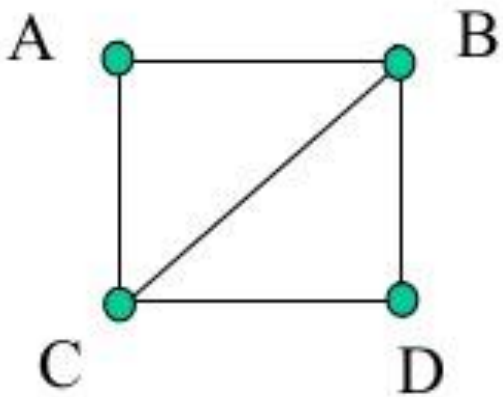
It is not simple.



It is a simple graph.

Adjacent, neighbors

- Two vertices are *adjacent* and are *neighbors* if they are the endpoints of an edge
- Example:
  - *A* and *B* are adjacent
  - *A* and *D* are not adjacent

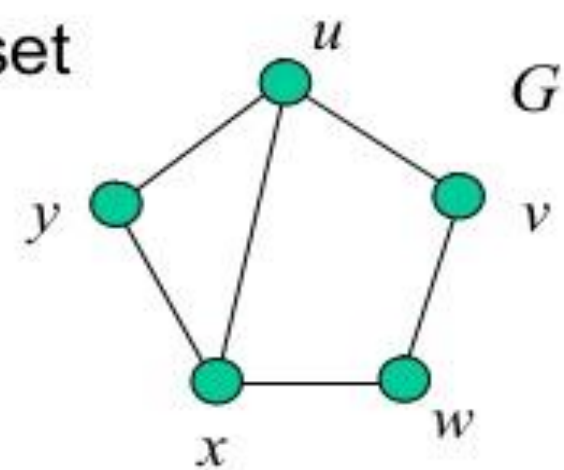


Finite Graph, Null Graph

- *Finite graph* : an graph whose vertex set and edge set are finite
- *Null graph* : the graph whose vertex set and edges are empty

Clique and Independent set

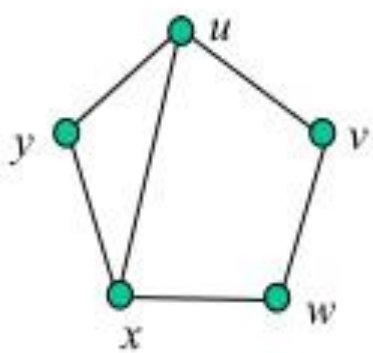
- A ***Clique*** in a graph: a set of pairwise adjacent vertices (a complete subgraph)
- An ***independent set*** in a graph: a set of pairwise nonadjacent vertices
- Example:
  - $\{x, y, u\}$ is a clique in G
  - $\{u, w\}$ is an independent set



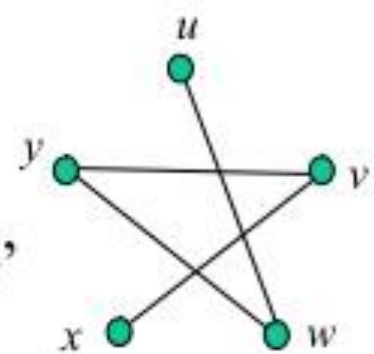
Complement

- ***Complement of G***: The complement G' of a simple graph G :
  - A simple graph
  - $V(G') = V(G)$
  - $E(G') = \{ uv \mid uv \notin E(G) \}$

G

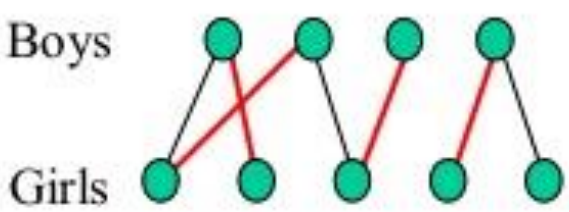


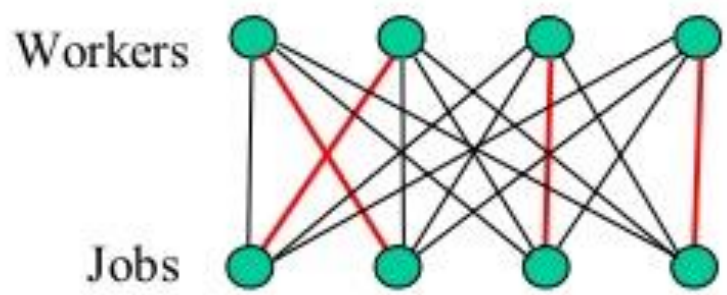
G'



Bipartite Graphs

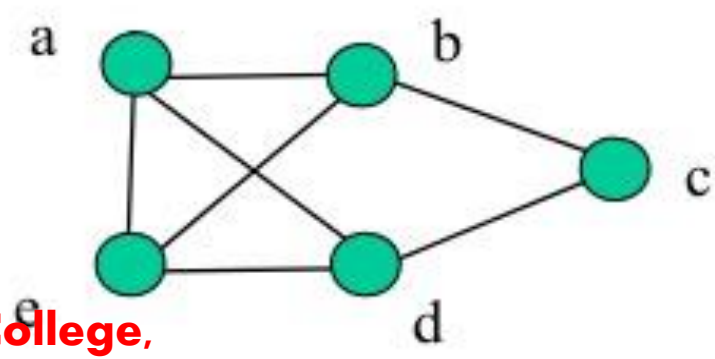
- A graph G is ***bipartite*** if $V(G)$ is the union of two disjoint independent sets called ***partite sets of*** G
- Also: The vertices can be partitioned into two sets such that each set is independent
- Matching Problem
- Job Assignment Problem





Path and Cycle

- ***Path*** : a sequence of distinct vertices such that two consecutive vertices are adjacent
  - Example: (a, d, c, b, e) is a path
  - (a, b, e, d, c, b, e, d) is not a path; it is a walk
- ***Cycle*** : a closed Path
  - Example: (a, d, c, b, e, a) is a cycle

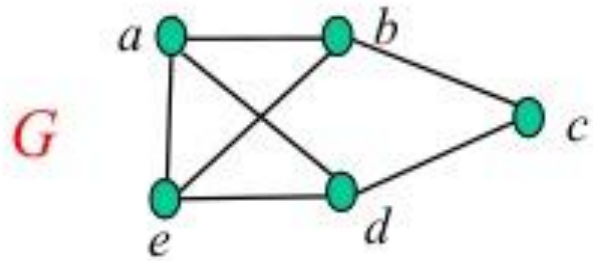


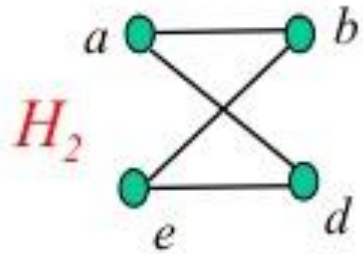
Subgraphs

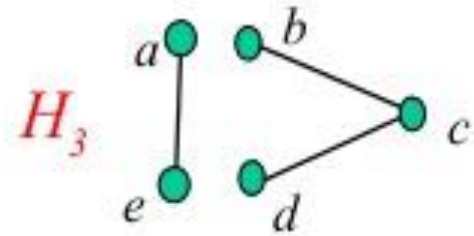
- A ***subgraph*** of a graph G is a graph H such that:
  - $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ and
  - The assignment of endpoints to edges in H is the same as in G.

Subgraphs

- Example: H_1, H_2, and H_3 are subgraphs of G

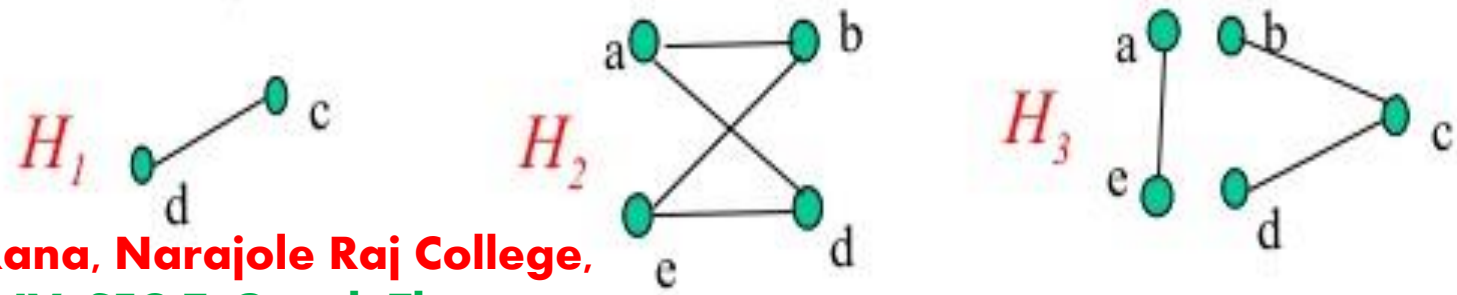






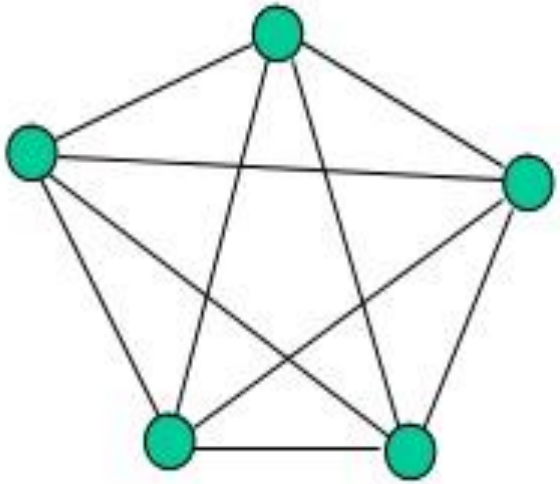
Connected and Disconnected

- ***Connected*** : There exists at least one path between two vertices
- ***Disconnected*** : Otherwise
- Example:
  - H_1 and H_2 are connected
  - H_3 is disconnected



Complete Graph

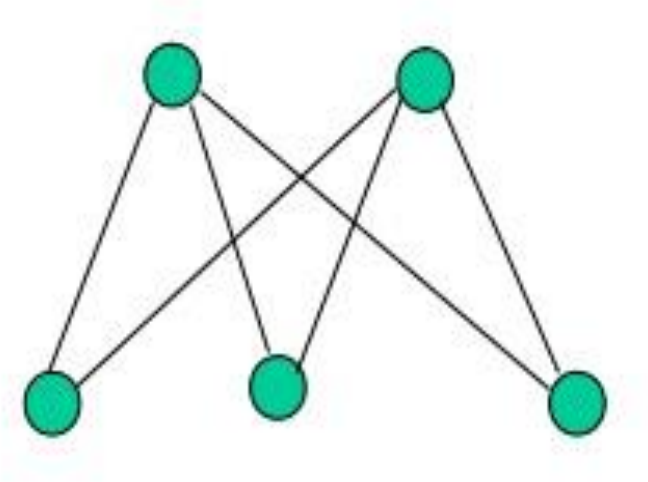
- ***Complete Graph*** : a simple graph whose vertices are pairwise adjacent



Complete Graph

Complete Bipartite Graph or Biclique

- ***Complete bipartite graph*** (biclique) is a simple bipartite graph such that two vertices are adjacent if and only if they are in different partite sets.



Complete Bipartite Graph

Thank You