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C13T (Electromagnetic Theory)

Topic – EM Wave in Bounded Media (Part – 1)

Introduction:

A wave is a disturbance of a continuous medium that propagates with a fixed shape at a constant velocity. It has an oscillatory nature with respect to both space and time. A wave also carries some form of energy when it propagates. Light is a form of electromagnetic energy, which flows like a transverse wave (with a speed c in vacuum). We call it an electromagnetic wave (EM wave), which is governed by *Maxwell's Equations*.

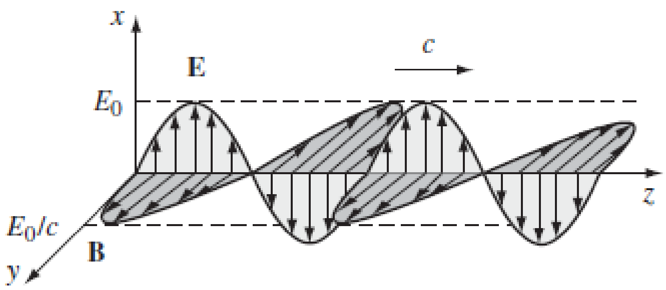

Fig. 1

If the electric field $\vec{E}$ points in the x direction, then the magnetic field $\vec{B}$ points in the y direction, we can write

$$\vec{E}(z,t) = E_0 e^{i(kz-\omega t)}\hat{x} \text{ and } \vec{B}(z,t) = \frac{1}{c} E_0 e^{i(kz-\omega t)}\hat{y}$$

This is the paradigm for a monochromatic plane wave (Fig. 1). The wave as a whole is said to be polarized in the x direction (by convention, we use the direction of $\vec{E}$ to specify the polarization of an electromagnetic wave). There is nothing special about the z direction, of course—we can easily generalize to monochromatic plane waves travelling in an arbitrary direction. The notation is facilitated by the introduction of the *propagation vector* (or *wave vector*) $\vec{k}$, pointing in the direction of propagation, whose magnitude is the wave number k. So, in general, it can be written as

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$$\vec{E}(\vec{r},t) = E_0 e^{i(\vec{k}.\vec{r}-\omega t)}\hat{n}$$

$$\vec{B}(\vec{r},t) = \frac{1}{c}E_0 e^{i(\vec{k}.\vec{r}-\omega t)}(\hat{k}\times\hat{n}) = \frac{1}{c}(\hat{k}\times\vec{E})$$

EM Wave Propagation in a Linear Medium:

In regions of space or vacuum where there is no charge or current, Maxwell's equations can be written as

$$\nabla.\vec{E} = 0$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla.\vec{B} = 0$$

$$\nabla\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

All these 4 equations give rise to these two final equations

$$\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \text{ and } \nabla^2\vec{B} = \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

These are nothing but the *wave equations*, where the velocity is given as $v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$, which again proves that light propagates with a speed c in vacuum. ε_0 and μ_0 are the absolute electric permittivity and absolute magnetic permeability.

Inside a medium, but in regions where there is no free charge or free current, Maxwell's equations become

$$\nabla.\vec{D} = 0$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla.\vec{B} = 0$$

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$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

For a linear medium $\vec{D} = \varepsilon \vec{E}$, and $\vec{H} = \frac{1}{\mu}\vec{B}$, where ε and μ are the electric permittivity and magnetic permeability of the medium respectively. Moreover, our assumption will be for a homogeneous medium as well. Therefore, ε and μ will be independent both of space and time. In that case, Maxwell's Equations reduce to

$$\nabla . \vec{E} = 0 \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla . \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu\varepsilon \frac{\partial \vec{E}}{\partial t}$$

Therefore, these equations will finally reduce to

$$\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \nabla^2 \vec{B} = \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}$$

Therefore inside a medium, light or EM wave travels with a speed

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{n} \text{ with } n = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$$

where n is called as the *refractive index* of the medium. Obviously, for vacuum $n = 1$.

For most of the material $\mu \approx \mu_0$. Therefore $n \approx \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{\varepsilon_r}$, with ε_r as the *relative permittivity* or *dielectric constant* of the medium.

Electrodynamic Boundary Conditions:

It is interesting to notice what happens when an EM wave passes from one transparent medium into another—air to water, say, or glass to plastic? The details depend on the exact nature of the *electrodynamic boundary conditions*, expressed as

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$$\varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp} \qquad \vec{E}_1^{\parallel} = \vec{E}_2^{\parallel}$$

$$B_1^{\perp} = B_2^{\perp} \qquad \frac{1}{\mu_1}\vec{B}_1^{\parallel} = \frac{1}{\mu_2}\vec{B}_2^{\parallel}$$

where the symbols ⊥ and || have been used to signify the perpendicular (normal) and parallel components. These boundary conditions will be used to look into the wave characteristics at the interface.

Reflection and Transmission with Oblique Incidence at the Interface between two Media:

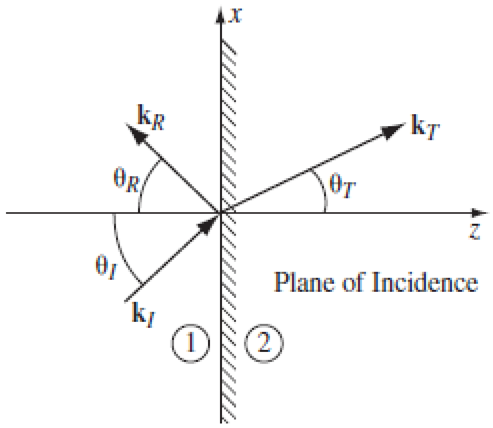

Fig. 2

Here the incoming wave meets the boundary at an arbitrary angle θ_I (Fig. 2). Of course, the normal incidence is really just a special case of oblique incidence, with $\theta_I = 0$. (In our discussion, the subscripts I, R and T will represent the parameters related to incident, reflected and transmitted (or refracted) beams.) θ_R and θ_T are the angles of reflection and refraction. The interface is described by $z = 0$ (or $x - y$ plane).

Let us suppose, that a monochromatic plane wave is represented as

$$\vec{E}_I(\vec{r}, t) = \vec{E}_{0I} e^{i(\vec{k}_I.\vec{r} - \omega t)} \text{ and } \vec{B}_I(\vec{r}, t) = \frac{1}{v_1}(\hat{k}_I \times \vec{E}_I)$$

This wave if approaching from the left, giving rise to a reflected wave and a transmitted wave given as (shown in Fig. 2)

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$$\vec{E}_R(\vec{r},t) = \vec{E}_{0R}e^{i(\vec{k}_R.\vec{r}-\omega t)} \text{ and } \vec{B}_R(\vec{r},t) = \frac{1}{v_1}(\hat{k}_R \times \vec{E}_R)$$

$$\vec{E}_T(\vec{r},t) = \vec{E}_{0T}e^{i(\vec{k}_T.\vec{r}-\omega t)} \text{ and } \vec{B}_T(\vec{r},t) = \frac{1}{v_2}(\hat{k}_T \times \vec{E}_T)$$

v_1 is the velocity of the wave in the medium 1 and v_2 is that inside the medium 2. The incident wave and reflected wave will be having the same velocity v_1 as they are confined in the same medium 1. But all three waves have the same frequency ω—that is determined once and for all at the source (the flashlight, or whatever, that produces the incident beam) and therefore $k_I v_1 = k_R v_1 = k_T v_2 = \omega$ or $k_I = k_R = \frac{v_2}{v_1} k_T = \frac{n_1}{n_2} k_T$.

Because the boundary conditions must hold at all points on the plane and for all times, the exponential factors in the electric and magnetic field terms must be equal (when $z = 0$).

Therefore, at $z = 0$, $\vec{k}_I.\vec{r} = \vec{k}_R.\vec{r} = \vec{k}_T.\vec{r}$, from which we obtain

$(k_I)_x = (k_R)_x = (k_T)_x$ at $y = 0$ and $(k_I)_y = (k_R)_y = (k_T)_y$ at $x = 0$

Proof of 1st Law of Reflection and Refraction:

If we assume that the incident beam was confined in $x - z$ plane, then $\vec{k}_I$ does not have any y −component, i.e. $(k_I)_y = 0$. We immediately get that both $\vec{k}_R$ and $\vec{k}_T$ do not have y −components either, so they are also confined in the same plane. This is nothing but the 1st law of reflection as well as refraction, which states that *the incident wave, reflected wave, refracted wave and the normal to the interface (here the z −axis) lie on the same plane*.

Proof of 2nd Law of Reflection:

From the identity $(k_I)_x = (k_R)_x = (k_T)_x$ we get $(k_I)_x = (k_R)_x$ or $k_I \sin\theta_I = k_R \sin\theta_R$. Since $k_I = k_R$, we get $\sin\theta_I = \sin\theta_R$ or $\theta_I = \theta_R$, which establishes the 2nd law of reflection, given as *the angle of incidence is equal to the angle of reflection*.

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Proof of 2nd Law of Refraction (Snell's Law):

Again from the identity $(k_I)_x = (k_R)_x = (k_T)_x$ we get $(k_I)_x = (k_T)_x$ or $k_I \sin\theta_I = k_T \sin\theta_T$ or $\frac{\sin\theta_I}{\sin\theta_T} = \frac{k_T}{k_I}$. Since $k_I = \frac{n_1}{n_2} k_T$, we obtain $\frac{\sin\theta_I}{\sin\theta_T} = \frac{n_2}{n_1}$ which establishes the 2nd law of refraction, stated as $\frac{\sin\theta_I}{\sin\theta_T} = \frac{n_2}{n_1}$, also popularly known as the *Snell's Law*.

This concludes part 1 of this e-report.

The discussion will be continuing in the part 2 of this e-report.

Reference:

Introduction to Electrodynamics, D.J. Griffiths, Pearson

(All the figures have been collected from the above mentioned reference)