COMPILED & CIRCULATED BY
Dr. Tapanendu Kamilya
Assistant Professor, Department of Physics, Narajole Raj College

Topic:

Wave Guides: Planar Optical Wave Guides. Planar Dielectric Wave Guide. Modal Analysis of EM wave in Planer Waveguide.

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WAVEGUIDE

1. Planar Optical Waveguide

An optical waveguide is a physical structure that guides electromagnetic waves in the optical spectrum. The waveguides having planar geometry and are capable of guide light only in one dimension is called planner waveguide. Planar waveguide are fabricated by a thin transparent film having increased refractive index on some substrate, or possibly embedded between two substrate layers. Planar waveguides are sometimes used for optical *amplifiers* with high gain as well as high power and planar waveguide lasers.

2. **Planar Dielectric Waveguide**

$n_1>n_2$

If a slab of dielectric material, called film or core, surrounded by media of lower refractive indexes, called cover and subtract as the upper and lower, respectively, that configures the dielectric waveguide. Here, the width of the slab is d and refraction index is n_1, and the cover and subtract have same refraction index n_2.) A light ray can be guided inside the slab by total internal reflection in the zigzag fashion. Only certain reflection angle θ will constructively interfere in the waveguide and hence only certain waves can exist in the waveguide.

3. Modal Analysis of Electromagnetic waves in planar waveguide

We consider a rectangular waveguide with air as dielectric.

Maxwell's equation-

$\nabla \times \mathbf{H}=\varepsilon_0 \frac{\partial E}{\partial t}$……………(1.8)

$\nabla \times \mathbf{E}=-\mu_0 \frac{\partial H}{\partial t}$…………………(1.9)

$\nabla . \mathbf{E}=0$………………………..(1.10)

$\nabla . \mathbf{H}=0$……………………(1.11)

If, the wave propagate with time impedance $e^{-i\omega t}$

Therefore, $\nabla \times \mathbf{H}=-i\omega\varepsilon_0 \boldsymbol{E}$…………(1.12)

$\nabla \times \mathbf{E}=i\omega\mu_0 \boldsymbol{H}$…………(1.13)

Taking curl of eq.-1.13 and by applying eq.-1.12

$(\nabla^2 + \frac{\omega^2}{c^2})\mathbf{E}=0$…………..(1.14)

The solution becomes

$\mathbf{E}(x,y,z,t)=E_0(x,y)e^{i(kz-wt)}$………………..(1.15i)

$\mathbf{H}(x,y,z,t)=H_0(x,y)e^{i(kz-wt)}$………………..(1.15ii)

Component analysis of Equn. 1.12 becomes-

$\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z} = -i\omega\varepsilon_0 E_x$

$\frac{\partial H_x}{\partial z}-\frac{\partial H_z}{\partial x} = -i\omega\varepsilon_0 E_y$

$\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y} = -i\omega\varepsilon_0 E_z$

By using sol^n (1.15)-

$\frac{\partial H_{oz}}{\partial y}-ik\, H_{oy} = -i\omega\varepsilon_0 E_{ox}$…………………………(1.16)

$ik\, H_{ox}-\frac{\partial H_{oz}}{\partial x}= -i\omega\varepsilon_0 E_{oy}$…………………..(1.17)

$\frac{\partial H_{oy}}{\partial x}-\frac{\partial H_{ox}}{\partial y} = -i\omega\varepsilon_0 E_{oz}$………………………(1.18)

Also, from equn.-(1.13) we get,

$\frac{\partial E_{oz}}{\partial y}-ik\, E_{oy} = i\omega\mu_0 H_{ox}$………………………(1.19)

$ik\, E_{ox}-\frac{\partial E_{oz}}{\partial x}= i\omega\mu_0 H_{oy}$…………………..(1.20)

$\frac{\partial E_{oy}}{\partial x}-\frac{\partial E_{ox}}{\partial y} = i\omega\mu_0 H_{oz}$………………………(1.21)

Eliminating E_{oy} from equn 1.17 and 1.19

$H_{ox}=\frac{i}{h^2}\,[k\frac{\partial H_{oz}}{\partial x} -\omega\varepsilon_0 \frac{\partial E_{oz}}{\partial y}]$ ………………………(1.22)

$H_{oy}=\frac{i}{h^2}\,[k\frac{\partial H_{oz}}{\partial y} +\omega\varepsilon_0 \frac{\partial E_{oz}}{\partial x}]$ ………………………(1.23)

$E_{ox}=\frac{i}{h^2}[k\frac{\partial H_{oz}}{\partial x}+\omega\mu_0\frac{\partial H_{oz}}{\partial y}]$ ....(1.24)

$E_{oy}=\frac{i}{h^2}[k\frac{\partial E_{oz}}{\partial x}-\omega\mu_0\frac{\partial H_{oz}}{\partial y}]$ ....(1.25)

H-waves: H waves are called when $E_{oz}=0$, Therefore, entire field is determined by H_{oz}

TE waves: when transverse electric field exists, then wave is called TE wave.

E-wave: E waves are called when $H_{oz}=0$, Therefore, entire field is determined by E_{oz}

TM waves: when transverse magnetic field exists, then wave is called TM wave.

When, $E_{oz}=0$ and $H_{oz}=0$, the wave is TEM wave (transverse electromagnetic wave). Pure TEM waves does not exist in rectangular wave guide.

TE mode:

$(\nabla^2+\frac{\omega^2}{c^2})\mathbf{H}=0$

$(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+h^2)Hoz=0$................................(1.26)

Let, $H_{oz}(x,y)=F_1(x).F_2(y)$ be the solution

From equation 19, by solving separation of variable,

$\frac{1}{F_1}\frac{d^2F_1}{dx^2}+\frac{1}{F_2}\frac{d^2F_2}{dx^2}=-h^2$

Taking, $\frac{1}{F_1}\frac{d^2F_1}{dx^2}=-k_x^2$ *and* $\frac{1}{F_2}\frac{d^2F_2}{dx^2}=-k_y^2$

The solution becomes, $H_{oz}(x,y)=(A\ Cosk_xx+B\ Sink_xx)(C\ Cosk_yy+D\ Sink_yy)$.......(1.27)

For, a rectangular wave guide,

$E_y=0$ at $x=0$ and $x=a$

$E_x=0$ at $y=0$ and $y=b$

Therefore, we get

$E_{oy} \propto \frac{\partial H_{oz}}{\partial x} = 0$ at $x = 0$ from equ^n 18 and B= 0 from equn -1.27

Also, $E_{ox} \propto \frac{\partial H_{oz}}{\partial y} = 0$ at $y = 0$ from equ^n 18 and D= 0 from equn -1.27

As a result, H_{oz} $(x,y) = H_o \, Cos \, k_x x \, Cos \, k_y y$, …….(1.28), *where* $H_o = AC$

Also, $E_{oy} \propto \frac{\partial H_{oz}}{\partial x} = 0$ at $x = a$, which specifies $Sin \, k_x a = 0$ or $k_x = \left(\frac{m\pi}{a}\right)$, where, m= 0,1,2,3,.. etc.

Similarly,

$E_{ox} \propto \frac{\partial H_{oz}}{\partial y} = 0$ at $y = b$, which specifies $Sin \, k_y b = 0$ or $k_y = \left(\frac{n\pi}{b}\right)$, where, n= 0,1,2,3,.. etc.

Therefore,

$$H_{oz} = H_o \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \qquad (1.29)$$

Also,

$$h^2 = k_x^{\,2} + k_y^{\,2}$$

Or,

$$\frac{\omega^2}{c^2} - k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$k^2 = \frac{\omega^2}{c^2} - \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \qquad (1.30)$$

If, $\omega < c\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$

As a result, the fields becomes attenuated and unable to propagate through waveguide, as, k becomes imaginary. Moreover, when

$$\omega > c\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2},$$

the fields becomes propagate through the waveguide, as, k becomes real.

Hence, here we can find a critical frequency (ω_c), which defines the frequency below which the frequency component cannot propagate through waveguides.

$$\omega_c = c\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \qquad (1.31)$$

And the cut off wavelength becomes,

$$\lambda_c = \frac{c}{\omega_c/2\pi} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} \qquad (1.32)$$

We know that (from equation 1.30 and 1.31)

$$k^2 = \frac{\omega^2}{c^2} - \frac{\omega_c{}^2}{c^2} \qquad (1.33)$$

Therefore, the guide wavelength becomes,

$$\lambda_g = \frac{2\pi}{k}$$

We know that, for free space wavelength $\lambda = \frac{c}{\omega/2\pi}$

Therefore,

$$\frac{1}{\lambda_g{}^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c{}^2}$$

$$\text{or,} \lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}} \qquad (1.34)$$

Therefore, the phase velocity

$$v_p = \frac{\omega}{k} = \frac{\omega}{\sqrt{\frac{\omega^2}{c^2} - \frac{\omega_c{}^2}{c^2}}} = \frac{c}{\sqrt{1-(\lambda/\lambda_c)^2}} \qquad (1.35)$$

TM modes:

For, **TM** modes,

$$H_{oz} = 0 \qquad (1.36)$$

Therefore, $E_{oz} = E_o \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$ (1.37)

Herein, the field component vanishes if both m and n or any one of them becomes zero. Hence, $(\mathbf{TM})_{10}$, $(\mathbf{TM})_{00}$, $(\mathbf{TM})_{01}$ modes cannot be present.

Finally, the highest wavelength for **TM** mode accessible transmission through waveguide becomes-

$$(\lambda_c)_{11} = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}}$$

Also, the highest wavelength for **TE** mode accessible transmission through waveguide becomes-

$$(\lambda_c)_{10} = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2}} \qquad assuming a > b$$

Therefore, the frequency band available for transmission through the waveguide is greater in **TE** modes than **TM** mode.

4. Summary

An optical waveguide is a physical structure that guides electromagnetic waves in the optical spectrum. The waveguides having planar geometry and are capable of guide light only in one dimension is called planner waveguide. Planar waveguide are fabricated by a thin transparent film having increased refractive index on some substrate, or possibly embedded between two substrate layers.TE waves: when transverse electric field exists, then wave is called TE wave. E-wave: E waves are called when H_{oz} =0, Therefore, entire field is determined by E_{oz}. TM waves: when transverse magnetic field exists, then wave is called TM wave.

Frequently Asked Questions (FAQ)

i) *What is Planner optical waveguide?*
ii) *What is planner dielectric waveguide?*
iii) *By modal analysis of EM wave in rectangular waveguide explain TE & TM modehence calculate the phase velocity and cut off wavelength.*

Numerical

i) *Calculate the cut off wavelength of a rectangular waveguide with 5.6 cm x 7.8 cm for (TE)10 mode. Hence, calculate the vg at frequency 12 GHz.*

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Dr. Tapanendu Kamilya
Assistant Professor, Department of Physics, Narajole Raj College

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