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C13T (Electromagnetic Theory)

Topic – Polarization of Electromagnetic Waves (Part – 1)

Introduction:

Polarization is a property applying to transverse waves that specifies the geometrical orientation of the oscillations of the wave. An electromagnetic or EM wave such as light consists of a coupled oscillating electric field and magnetic field which are always perpendicular to each other. By convention, the polarization of electromagnetic waves refers to the direction of the electric field. It is an important parameter in areas of science dealing with transverse waves, such as optics, seismology, radio, and microwaves. Especially impacted are technologies such as lasers, wireless and optical fibre telecommunications and radar. The simplest electromagnetic field type is that of a plane wave. Then each Cartesian component of the field (electric as well as magnetic) vectors and consequently the vectors $\vec{E}$ and $\vec{H}$ themselves are functions of the variable $\tau = \omega t - \vec{k}.\vec{r}$ only.

Different types of Polarization:

Of particular interest is the case when the plane wave is time-harmonic, i.e. when each Cartesian component of $\vec{E}$ and $\vec{H}$ is of the form

$$a\cos(\tau + \delta) = Re\{ae^{-i(\tau+\delta)}\} \text{ with } a > 0$$

Here τ denotes the variable part of the phase factor, i.e. $\tau = \omega t - \vec{k}.\vec{r}$

Now we discuss different types of polarization. We choose the $z-$axis in the propagation direction. Then, since the field is transverse, only the x and y components of $\vec{E}$ and $\vec{H}$ will be non-zero. We shall now consider the nature of the curve which the end point of the electric vector describes at a typical point in space. This curve is the locus of the points whose coordinates (E_x, E_y) are given by

$$E_x = a_1\cos(\tau + \delta_1)$$

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$$E_y = a_2 \cos(\tau + \delta_2)$$

Elliptic Polarization. In order to eliminate τ between the first two equations of, we re-write them in the form

$$\frac{E_x}{a_1} = \cos(\tau + \delta_1) = \cos\tau\cos\delta_1 - \sin\tau\sin\delta_1$$

$$\frac{E_y}{a_2} = \cos(\tau + \delta_2) = \cos\tau\cos\delta_2 - \sin\tau\sin\delta_2$$

After a little bit of algebra we obtain

$$\left(\frac{E_x}{a_1}\right)^2 + \left(\frac{E_y}{a_2}\right)^2 - 2\left(\frac{E_x}{a_1}\right)\left(\frac{E_y}{a_2}\right)\cos\delta = \sin^2\delta$$

where $\delta = \delta_2 - \delta_1$. This gives the equation of an ellipse. The ellipse is inscribed in a rectangle whose sides are parallel to the coordinate axes and whose lengths are $2a_1$ and $2a_2$ (as shown in Fig. 1). The ellipse touches the sides at the point $(\pm a_1, \pm a_2\cos\delta)$ and $(\pm a_1\cos\delta, \pm a_2)$.

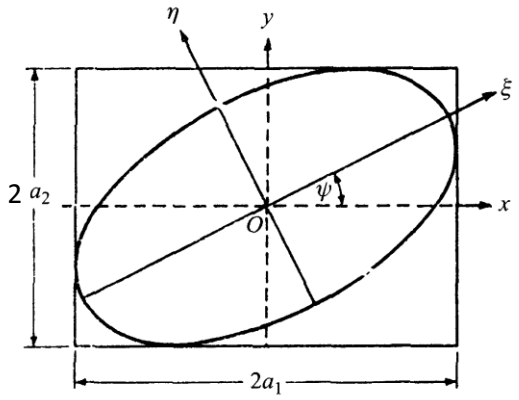

Fig. 1

The wave in this case is said to be *elliptically polarized*. In general the axes of the ellipse are not in the x and y directions. Let ξ and η be a new set of axes along the axes of the ellipse and let ψ be the angle between x and the direction ξ of the major axis (Fig. 1). Then the components E_ξ and E_η are related to E_x and E_y by

$$E_\xi = E_x\cos\psi + E_y\sin\psi$$

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$$E_\eta = -E_x \sin\psi + E_y \cos\psi$$

We distinguish two cases of polarization, according to the sense in which the end point of the electric vector describes the ellipse. It seems natural to call the polarization right-handed or left-handed according to whether the rotation of $\overrightarrow{E}$ and the direction of propagation form a right-handed or left-handed screw. But the traditional terminology is just the opposite — being based on the apparent behaviour of $\overrightarrow{E}$ when 'viewed' face on by the observer. Thus we say that the polarization is *right-handed* when to an observer looking in the direction from which the light is coming, the end point of the electric vector would appear to describe the ellipse in the *clockwise* sense. We see that in this case $\sin\delta > 0$. For *left-handed* polarization the opposite is the case, i.e. to an observer looking in the direction from which the light is propagated, the electric vector would appear to describe the ellipse *anticlockwise* and in this case $\sin\delta < 0$.

Linear Polarization and Circular Polarization. Two special cases are of particular importance, namely when the polarization ellipse degenerates into a straight line or a circle.

The ellipse will reduce to a straight line when $\delta = \delta_2 - \delta_1 = m\pi$ where m is any positive or negative integer. There we obtain $\frac{E_y}{E_x} = (-1)^m \frac{a_2}{a_1}$, which is an equation of a straight line. We say that the wave is *linearly polarized* (or less frequently *plane polarized*).

The other special case of importance is that of a *circularly polarized* wave, the ellipse then degenerating into a circle. Clearly a necessary condition for this is that the circumscribed rectangle shall become a square with sides $a_1 = a_2 = a$. And also it is required that $\delta = \delta_2 - \delta_1 = \frac{m\pi}{2}$ with m any positive or negative odd integer. Here we obtain the equation of a circle given by ${E_x}^2 + {E_y}^2 = a^2$.

If instead of the real representation, the complex one is used (i.e. if the exponential instead of the cosine function is written), then

$$\frac{E_y}{E_x} = \frac{a_2}{a_1} e^{-i\delta}$$

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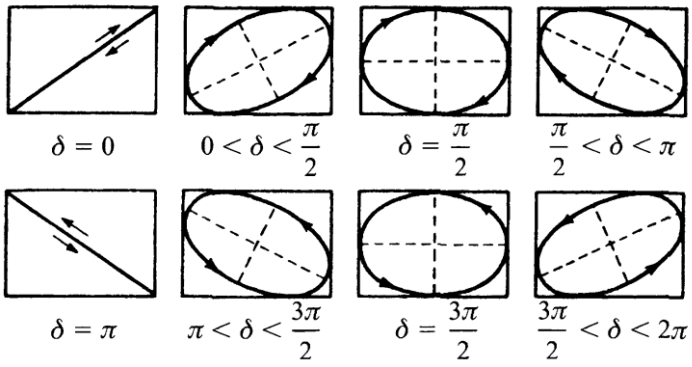

Fig. 2

From the value of this ratio one can immediately determine the nature of the polarization. For example, we obtain

(i) Linearly polarized electric wave ($\delta = m\pi, m = 0, \pm 1, \pm 2, ...$)

$$\frac{E_y}{E_x} = (-1)^m \frac{a_2}{a_1}$$

(ii) Right-handed circularly polarized electric wave ($a_1 = a_2, \delta = \frac{m\pi}{2}$)

$$\frac{E_y}{E_x} = e^{-\frac{im\pi}{2}} = -i$$

(iii) Left-handed circularly polarized electric wave ($a_1 = a_2, \delta = -\frac{m\pi}{2}$)

$$\frac{E_y}{E_x} = e^{\frac{im\pi}{2}} = i$$

Fig. 2 illustrates how the polarization ellipse changes with varying δ.

Discussion on Anisotropic Media, Dielectric Tensor:

It is to be remembered that the optical theory is based on two quite separate foundations, on the one hand on Maxwell's equations and on the other hand on the material equations which in the case of an *isotropic medium* were given. In dealing with crystals we must generalize these latter equations so as to take account of *anisotropy*. In this e-report we assume that the medium is homogeneous, non-conducting ($\sigma = 0$) and magnetically isotropic, but allow

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electrical anisotropy, i.e. we consider substances whose electrical excitations depend on the direction of the electric field. In general the displacement vector $\vec{D}$ will then no longer be in the direction of the vector $\vec{E}$. We have to write each component of $\vec{D}$ by the following (instead of $\vec{D} = \varepsilon\vec{E}$)

$$D_x = \varepsilon_{xx}E_x + \varepsilon_{xy}E_y + \varepsilon_{xz}E_z$$

$$D_y = \varepsilon_{yx}E_x + \varepsilon_{yy}E_y + \varepsilon_{yz}E_z$$

$$D_z = \varepsilon_{zx}E_x + \varepsilon_{zy}E_y + \varepsilon_{zz}E_z$$

The nine quantities ε_{xx}, ε_{yy} etc. are constants of the medium, and constitute a tensor, called the *dielectric tensor*. The vector $\vec{D}$ is thus the product of this tensor with $\vec{E}$.

Symmetric Nature of Dielectric Tensor. We can write all the 3 equations by

$$D_k = \sum_l \varepsilon_{kl}E_l$$

where k stands for one of the three indices x, y and z, and l stands for each of x, y and z in turn in the summation.

The expressions of electric and magnetic energy densities are written as

$$w_e = \frac{1}{2}\vec{E}.\vec{D} = \frac{1}{2}\sum_{kl} E_k\varepsilon_{kl}E_l$$

$$w_m = \frac{1}{2}\vec{B}.\vec{H} = \frac{1}{2}\mu H^2$$

Now from Maxwell's equations, it can shown using elementary vector algebra that $-\nabla.\left(\vec{E}\times\vec{H}\right) = \vec{E}.\dot{\vec{D}} + \vec{H}.\dot{\vec{B}} = \sum_{kl} E_k\varepsilon_{kl}\dot{E}_l + \frac{1}{2}\frac{d}{dt}(\mu H^2)$

The second term on the right represents the rate of change of the magnetic energy density $\frac{dw_m}{dt}$, but the first term does not represent the rate of change of the electric energy density unless $\sum_{kl} E_k\varepsilon_{kl}\dot{E}_l = \frac{dw_e}{dt} = \frac{1}{2}\sum_{kl}\varepsilon_{kl}(E_k\dot{E}_l + E_l\dot{E}_k)$ or unless $\sum_{kl}\varepsilon_{kl}(E_k\dot{E}_l - E_l\dot{E}_k) = 0$.

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The suffixes k and l are dummy suffixes; both run over the same values (x, y, z). Hence the expression is not altered if we interchange k and l in the second term. This leads to $\sum_{kl}(\varepsilon_{kl} - \varepsilon_{lk})E_k\dot{E}_l = 0$, which is possible only if $\varepsilon_{kl} - \varepsilon_{lk} = 0$ or $\varepsilon_{kl} = \varepsilon_{lk}$. This means that the dielectric tensor must be *symmetric*. It has only six instead of nine independent components.

This concludes part 1 of this e-report.

The discussion will be continuing in the part 2 of this e-report.

Reference:

Principles of Optics, Max Born & Emil Wolf, Cambridge University Press

(All the figures have been collected from the above mentioned reference)